1. Introduction

In many engineering structures, such as chemical and nuclear waste storage and oil boreholes, geomaterials are widely studied and used. For example, the hard clayey rock (argillite) has been extensively investigated in the context of feasibility study for geological disposal of radioactive wastes (e.g. Chiarelli, 2000; Andra, 2005; Robinet, 2008). Due to their low permeability, relatively high mechanical strength and the absence of major tectonic fractures, these clayey rocks are envisaged as one of potential geological barriers. Porous chalks and anisotropic sedimentary rocks have been extensively studied in mining engineering and petroleum industry (Donath, 1961; McLamore and Gray, 1967; Atwell and Sandford, 1974; Lerau et al., 1981; Hoek, 1983; Nianlou et al., 1997; Papargiriou et al., 1997; Homand and Shao, 2000; Schroeder, 2003; De Gennaro et al., 2004; Alam et al., 2010). Most geomaterials have complex microstructures (pores, mineral inclusions, microcracks, etc.) which induce complex macroscopic behaviors, such as mean stress dependency, microcrack-related damage, plastic deformation, volumetric compressibility and dilatancy, and dissymmetric responses between tensile and compressive stresses. Extensive experimental data showed that the pores or inclusions significantly affect the mechanical strength and deformation behaviors of heterogeneous materials. Different kinds of constitutive models have been proposed for modeling the effective behaviors of geomaterials. In most of them, phenomenological elastoplastic and damage models have been used for the macroscopic behaviors of Callovian-Oxfordian (COx) argillite (e.g. Chiarelli et al., 2003; Shao et al., 2006; Hoxha et al., 2007), or for anisotropic materials (Walsh and Brace, 1964; Pariseau, 1968; Hoek and Brown, 1980; Nova, 1980; Pietruszczak and Mroz, 2001; Pietruszczak et al., 2002; Lydzba et al., 2003; Lee and Pietruszczak, 2008). The formulation of models is essentially based on the standard framework of thermodynamics and experimental evidences. However, it is admitted that the mechanical behavior of geomaterials is inherently related to the composition and mechanical properties of constituents. The macroscopic response directly depends on the evolution of material microstructure such as change of porosity, microcrack initiation and growth, and physical and chemical reactions. The phenomenological models cannot properly consider such relationships between microstructure and macroscopic behaviors, for
example, the porosity and mineralogical compositions. In order to overcome this weakness of phenomenological models, micro-macro models have been developed during the last decades with different homogenization techniques. As an example of COx argillite, Abou-Chakra Guéry et al. (2008) proposed a meso-macro model for clayey rocks using the Hill's incremental approach. The claystone was considered as a three-phase composite constituted by a clay matrix, calcite and quartz grains. The clay matrix is described by a classical Drucker–Prager type plastic model and the porosity inside the clay matrix was neglected. This model was improved by Shen et al. (2012a, 2013a) by considering the porosity in the clay matrix which is described by a porous medium at the microscale. Recently, a micromechanical elastoplastic model was proposed in Shen and Shao (2015a) for anisotropic sedimentary rocks. The effects of porosity, inclusions and inherent anisotropy were explicitly taken into account. In many engineering applications, geomaterials are subjected not only to mechanical loading, but also to moisture transfer, variation of temperature and chemical degradation. The durability analysis of such structures requires the consideration of such multi-physical coupling phenomena. The elastic modulus and mechanical strength can significantly vary with the variation of water content during drying or wetting process. To this end, the effects of water saturation degree and chemical reaction carbonation are considered for cement-based materials in micromechanics-based models (Chen et al., 2013; Shen et al., 2015a).

In this study, we focus on the effects of pores and inclusions on the macroscopic mechanical behavior of porous geomaterials. Some micro-macro constitutive models will be presented to describe the elastoplastic behaviors of the studied material. The paper is organized as follows. In Section 2, some macroscopic criteria will be presented for ductile porous materials with one population of pore. Different types of matrices are considered (von Mises, Green type, Mises–Prager). For simplicity, the incompressible matrix that obeys the von Mises criterion:

\[ \sigma_{m} = \sigma_{eq} - \sigma_{0} \leq 0 \]  

where \( \sigma_{2}^{eq} = (3/2)\sigma' : \sigma' \) with \( \sigma' \) representing the deviatoric part of the microscopic stress tensor \( \sigma \), and \( \sigma_{0} \) is related to the shear strength.

There are many macroscopic criteria for porous media with a von Mises matrix. The micromechanics-based model proposed by Gurson (1977) is one of the most frequently used methods. Within the framework of kinematical limit analysis theory, a macroscopic criterion is obtained for a von Mises material containing a spherical or cylindrical void with a uniform macroscopic strain rate boundary condition:

\[ \phi = \frac{\Sigma_{eq}}{\sigma_{0}} - 2f \cosh \left( \frac{3}{2}\Sigma_{m} \right) - 1 - f^2 = 0 \]  

where \( \Sigma_{eq} \) and \( \Sigma_{m} \) denote the macroscopic equivalent stress and the macroscopic hydrostatic one, respectively.

Based on this famous model, a huge number of extensions have been proposed (e.g. considering the void shape effect (Gologanu et al., 1993, 1994, 1997; Pardoen and Hutchinson, 2000, 2003; Monchiet et al., 2008, 2014; Shen et al., 2011), taking into account the tension-compression asymmetry and the anisotropy of the matrix (Benzeraga et al., 1999; Monchiet et al., 2008; Cazacu and Stewart, 2009; Stewart and Cazacu, 2011). To better reproduce unit-cell simulations, Tvergaard (1981) and Tvergaard and Needleman (1984) (see also Tvergaard (1989)) proposed a heuristic and phenomenological extension of the Gurson model, known as the GTN model which is widely used in structural computations:

\[ \frac{\Sigma_{eq}^{2}}{\sigma_{0}^{2}} + 2q_{1} f \cosh \left( \frac{3}{2}q_{2} \Sigma_{m}^{2} \right) - 1 - q_{3} f^{2} = 0 \]  

where \( q_{1} \), \( q_{2} \) and \( q_{3} \) are the model parameters which have great influences on the yield surface.

On the other hand, Ponte Castaneda (1991) proposed a macroscopic criterion for an isotropic porous medium with a von Mises matrix by using a variational method:

\[ \left( 1 + \frac{2}{3} f \right) \left( \frac{\Sigma_{eq}}{\sigma_{0}} \right)^{2} + \frac{9}{4} f \left( \frac{\Sigma_{m}}{\sigma_{0}} \right)^{2} - \left( 1 - f \right)^{2} = 0 \]  

This criterion was improved in Michel and Suquet (1992) by modifying the term \( \Sigma_{m} \):

\[ \left( 1 + \frac{2}{3} f \right) \left( \frac{\Sigma_{eq}}{\sigma_{0}} \right)^{2} + \frac{9}{4} \left( 1 - f \right) \left( \ln f \right) \left( \frac{\Sigma_{m}}{\sigma_{0}} \right)^{2} - \left( 1 - f \right)^{2} = 0 \]  

2. Macroscopic criteria of ductile porous materials with one population of pores

In this section, some macroscopic criteria of ductile porous materials with one population of voids will be firstly presented. For this class of porous material, the representative volume element (RVE) is usually chosen as the one in Fig. 1, a hollow sphere with the internal and external radii respectively denoted \( a \) and \( b \). We denote \( V \) the volume of the unit cell, \( \langle V \rangle = 4\pi b^{3}/3 \), whereas \( \langle a \rangle \) the volume of the mesoscopic voids, \( \langle a \rangle = 4\pi a^{3}/3 \). The porosity corresponding to this hollow sphere is \( f = a^{3}/b^{3} \). Uniform strain rate boundary conditions are applied on the outer surface of the hollow sphere:

\[ \varepsilon = D \dot{\varepsilon} \quad \text{for} \quad x = b \varepsilon_{0} \]  

where \( D \) represents a uniform macroscopic strain rate. In Fig. 1, different types of matrices will be studied in this paper (von Mises, Green type, Mises–Schleicher and Drucker–Prager). For simplicity, the incompressible matrix that obeys the von Mises criterion will be firstly considered.

2.1. Porous medium with a von Mises type matrix

For a ductile porous material with one population of voids, the matrix in Fig. 1 is assumed to obey von Mises criterion:

\[ \phi = \frac{\Sigma_{eq}}{\sigma_{0}} - 2f \cosh \left( \frac{3}{2} \Sigma_{m} \right) - 1 - f^{2} = 0 \]  

where \( \Sigma_{eq} \) and \( \Sigma_{m} \) denote the macroscopic equivalent stress and the macroscopic hydrostatic one, respectively.

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\[ \frac{\Sigma_{eq}^{2}}{\sigma_{0}^{2}} + 2q_{1} f \cosh \left( \frac{3}{2}q_{2} \Sigma_{m}^{2} \right) - 1 - q_{3} f^{2} = 0 \]  

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This criterion was improved in Michel and Suquet (1992) by modifying the term \( \Sigma_{m} \):

\[ \left( 1 + \frac{2}{3} f \right) \left( \frac{\Sigma_{eq}}{\sigma_{0}} \right)^{2} + \frac{9}{4} \left( 1 - f \right) \left( \ln f \right) \left( \frac{\Sigma_{m}}{\sigma_{0}} \right)^{2} - \left( 1 - f \right)^{2} = 0 \]  

Fig. 1. Hollow sphere with uniform strain rate boundary conditions.
Recently, the Gurson’s criterion was improved by Cheng et al. (2014) and Shen et al. (2015b) using the stress variational homogenization (SVH). We know that the von Mises criterion is suitable for the metallic materials which are incompressible. This is not the case of geomaterial whose tensile behavior is very different from the compressive one. The plastic compression in the matrix should be taken into account. For this purpose, compressible matrix (Green type, Mises–Schleicher type and Drucker–Prager type) will be studied here.

2.2. Porous medium with a Green type matrix

As shown in Eq. (1), the von Mises criterion is independent of the hydrostatic stress \( \sigma_m \). In order to consider the compressibility of the matrix, the matrix in Fig. 1 is assumed to obey a general elliptic strain rate boundary conditions, the following inequality holds for all admissible with \( \sigma_m \) (Suquet, 1985; De Buhan, 1986):

\[
\Phi(\sigma) = \beta \sigma_{eq}^2 + \frac{9\alpha}{2} \sigma_m^2 - L \sigma_m - \sigma_0^2 \leq 0
\]

where \( \sigma_m \) is the mean stress of the microscopic stress \( \sigma \) in the matrix. Scalars \( \alpha, \beta \) and \( L \) are the material constants (a von Mises matrix is obtained for \( L = 0 \) together with \( \alpha = 0 \)). The yield function of Eq. (6) generalizes the one used in Shen et al. (2012b).

The limit analysis approach used in Gurson’s model will be adopted here to establish a macroscopic criterion for the studied porous medium having a Green type matrix. The local plastic dissipation \( \pi(\mathbf{d}) \), corresponding to Eq. (6), takes the following form:

\[
\pi(\mathbf{d}) = \frac{L}{3\alpha} \mathbf{d}_{eq} + \sqrt{\sigma_0^2 + \frac{L^2}{18\alpha}} \sqrt{\frac{2\sigma_m^2}{\alpha} + \frac{d_{eq}^2}{\beta}}
\]

where \( \mathbf{d}_{eq} = \sqrt{(2/3)\mathbf{d}^2} \mathbf{d} \) is the equivalent strain rate with \( \mathbf{d} \) representing the deviatoric part of the mesoscopic strain rate \( \mathbf{d} \), and \( d_{eq} = \mathbf{d} / 3 \) is the volumetric strain rate.

Considering the RVE presented in Fig. 1 with the uniform strain rate boundary conditions, the following inequality holds for all macroscopic stress \( \Sigma \) and macroscopic strain rate \( \mathbf{D} \) (Suquet, 1985; De Buhan, 1986):

\[
\Sigma : \mathbf{D} \leq \Pi(\mathbf{D}) = \inf_{\mathbf{D}} \int_{\Omega} \frac{1}{2} \pi(\mathbf{d}) \, dV
\]

where \( \Pi(\mathbf{D}) \) represents the macroscopic dissipation. The infimum in Eq. (8) is taken over all kinematically admissible (KA) velocity fields, \( \mathbf{v} \). Classically, the limit stress rates at the macroscopic scale are given by

\[
\mathbf{\Sigma} = \frac{\partial \Pi(\mathbf{D})}{\partial \mathbf{D}}
\]

The determination of \( \Pi(\mathbf{D}) \) requires the choice of a trial velocity field which must be plastically admissible. Based on the velocity used in Gurson’s model \( (\mathbf{u}^G) \), the following velocity field is chosen for the elliptic matrix:

\[
\mathbf{v} = \mathbf{A}_x + \mathbf{v}^G = \mathbf{A}_x + \frac{b^3(D_m - A)}{r^2} \mathbf{e}_r + \mathbf{D}_x
\]

where \( D_m = \text{tr} \mathbf{D} / 3 \), and \( \mathbf{D}_x \) is the deviatoric part of the macroscopic strain rate \( \mathbf{D} \). The homogenous field \( \mathbf{A}_x \) allows to account for matrix plastic compressibility. The two remaining terms (which are of Rice-Tracey type already considered by Gurson) are kinematically admissible with \( \mathbf{D} = \mathbf{A}_1 \) where \( 1 \) is the second-order identity tensor. More precisely, the second term on the right side of Eq. (10) corresponds to the expansion of the cavity and the outer volume, while the third one describes the shape change of the cavity and of the outer boundary without volume change. Hence, for any value of the scalar \( A \), the whole velocity field \( \mathbf{v} \) complies with the uniform strain rate \( \mathbf{D} \) applied to the hollow sphere.

Due to the presence of \( A \) (which remains unknown in the definition of the velocity field), the macroscopic dissipation, \( \Pi(\mathbf{D}) \), is computed owing to a minimization procedure with respect to \( A \):

\[
\Pi(\mathbf{D}) = \min_A \left[ \tilde{\Pi}(\mathbf{D}, A) \right]
\]

where

\[
\tilde{\Pi}(\mathbf{D}, A) = \left( \frac{\sigma_0^2 + \frac{L^2}{18\alpha}}{2} \right) \int_{\Omega} \sqrt{\frac{2\sigma_m^2}{\alpha} + \frac{d_{eq}^2}{\beta}} \, dV
\]

\[
+ \int_{a}^{b} \frac{L}{\Omega} (3\alpha) \int_{\Omega} \mathbf{d} \, dV
\]

The determination of the macroscopic criterion requires to compute the integrals of \( d_{eq} \) and \( d_m \) over the matrix (see Eq. (11)). The strain rate in the solid matrix can be obtained from Eq. (10), in spherical coordinates, as

\[
\mathbf{d} = \mathbf{A}_1 + \mathbf{D} + \frac{b^3(D_m - A)}{r^2} (1 - 3\mathbf{e}_r \otimes \mathbf{e}_r)
\]

In order to obtain a closed-form expression, the following inequality is classically used:

\[
\int_{\Omega} \pi(\mathbf{d}) \, dV \leq 4\pi \int_{a}^{b} \left( \frac{<\pi(\mathbf{d})>_S(r)}{r} \right)^{1/2} \, dr
\]

where \( S(r) \) is the sphere with the radius of \( r \), and \( <\pi(\mathbf{d})>_S(r) \) is the average of \( \pi(\mathbf{d}, r, \theta, \phi) \) over all the orientations.

According to Eqs. (11)–(13), the macroscopic dissipation \( \tilde{\Pi}(\mathbf{D}, A) \) can be computed:

\[
\tilde{\Pi}(\mathbf{D}, A) = \sqrt{\sigma_0^2 + \frac{L^2}{18\alpha}} \left[ \frac{\text{Narcsh}(n\mathbf{M}^N)}{\mathbf{M}^N} - \sqrt{M^2 + N^2u^2} \right]^{1/2}
\]

\[
+ \frac{L}{3\alpha} A (1 - f)
\]

where

\[
M^2 = \frac{2A^2}{\alpha} + \frac{D_{eq}^2}{\beta}, \quad N^2 = \frac{4}{3} (D_m - A)^2
\]

We need to minimize \( \tilde{\Pi}(\mathbf{D}, A) \) over the unknown variable \( A \) and to determine the macroscopic yield function by taking advantage of the approximate expression of \( \tilde{\Pi}(\mathbf{D}, A) \):

\[
\Sigma = \frac{\partial \tilde{\Pi}(\mathbf{D}, A)}{\partial \mathbf{D}}, \quad \frac{\partial \tilde{\Pi}(\mathbf{D}, A)}{\partial A} = 0
\]

Similarly to the approach used by Gurson, one can then establish the parametric form of the macroscopic yield function:
The macroscopic yield function (Eq. (17)) is of Gursen type with appropriate quantities $\Sigma_A$ and $\Sigma_B$ which need to be explicit. Noticing that $M$ depends only on the deviation $D$ and scalar $A$, while $N$ is a function of $D_m$ and $A$, the expressions of $\partial \Sigma / \partial M$ and $\partial \Sigma / \partial N$ can be calculated with the condition of minimization with respect to $A$.

As a final result, the closed-form expression of the macroscopic criterion for the porous medium having a matrix obeying the general elliptic criterion (Eq. (6)) is (more details about the calculation can be found in Shen et al. (2014a)):

$$\beta \frac{\Sigma_m^2}{\sigma_0^2 + L^2/(18\alpha)} + \frac{9\alpha}{2} \left[ \Sigma_m - L(1-\beta)/(9\alpha) \right]^2$$

$$+ 2f \cosh \left[ \frac{3\sqrt{\beta}}{2} \frac{\Sigma_m}{\sqrt{\sigma_0^2 + L^2/(18\alpha)}} \right] - 1 - f^2 = 0$$

(18)

The Gursen’s criterion can be obviously retrieved when $L = 0$, $\alpha = 0$ and $\beta = 1$. The macroscopic criterion (Eq. (18)) has been extended to hydromechanical case with saturated pore by using Eshelby-like velocity fields (Shen et al., 2013b, 2014b).

2.3. Porous medium with a Mises–Schleicher type matrix

For many polymers and geomaterials (Aubertin and Li, 2004), the strength difference between uniaxial tension and uniaxial compression can be represented by the Mises–Schleicher criterion (Schleicher, 1926). Many works have been done for this pressure-sensitive criterion (e.g. Raghava et al., 1973; Lubliner, 1990; Lee and Oung, 2000; Kovrizhnykh, 2004; Zhang et al., 2008; Durban et al., 2010). In this section, the isotropic and pressure-sensitive matrix is assumed to obey the Mises–Schleicher criterion:

$$f(\sigma) = \sigma_{eq}^2 + 3\alpha \sigma_0 \sigma_m - \sigma_0^2 \leq 0$$

(19)

Due to the compressibility in the matrix and the difficulty in finding a suitable kinematically admissible velocity field, the upper bound approach used in Section 2.2 cannot be adopted. A heuristic macroscopic criterion was established in Shen et al. (2015c) for this porous material by considering some special conditions.

(1) Case of purely hydrostatic loading

In the special case of purely hydrostatic loading ($\Sigma_{eq} = 0$), the exact values of $\Sigma_m/\sigma_0$ of the hollow sphere with a Mises–Schleicher compressible matrix have been obtained in Monchiet and Kondo (2012):

$$\frac{\Sigma_m}{\sigma_0} = 1 - \frac{a^2W_0(f_p^+)}{3\alpha} - 2a^2W_0(f_p^+)$$

$$p_+ = z_+ \exp(z_+)$$

$$z_+ = -\alpha + \sqrt{\alpha^2 + 1}$$

for hydrostatic tensile loading

$$\frac{\Sigma_m}{\sigma_0} = 1 - \frac{a^2W_1(f_p^-) - 2a^2W_1(f_p^-)}{3\alpha}$$

$$p_- = z_- \exp(z_-)$$

$$z_- = -\alpha - \sqrt{\alpha^2 + 1}$$

(20)

(21)

For hydrostatic compressive loading, where $W$ denotes the “Lambert W” function which satisfies $W(x)e^{W(x)} = x$. $W(x)$ has two branches: upper branch $W_0(x) \geq -1$ (for $x \geq -1/e$) and lower branch $W_{-1}(x) < -1$ (for $-1/e < x < 0$). In the case of tension, $W(px)$ in Eq. (21) is $W_0(px)$, whereas in the case of compression, $W(px) = W_{-1}(px)$.

For a general case, the exact solution of purely hydrostatic loading (Eq. (21)) can be rewritten as the following form (the value of $a^2W_0(f_p) + 2a^2W_0(f_p)$ is positive):

$$\left[a^2W_0(f_p) + 2a^2W_0(f_p)\right]^A = \exp[A \ln(1 - 3\alpha \Sigma_m/\sigma_0)]$$

(22)

In Eq. (22), the parameter $A$ does not affect the accuracy of the exact solution in the case of purely hydrostatic loading and will be determined in the second special case of an incompressible matrix.

(2) Case of the compressible parameter $\alpha \to 0$ and $\Sigma_{eq} = 0$

For an incompressible case ($\alpha \to 0$), the Mises–Schleicher criterion (Eq. (19)) reduces to the one of von Mises and there is non-compressibility in the matrix. The famous Gursen’s criterion (Eq. (2)) should be retrieved from the searched macroscopic criterion when $\alpha \to 0$. To this end, the following mathematical property is used for the appearance of the “$\cosh()$” term as the one in Gursen’s criterion:

$$\gamma = e^{-\phi} \Leftrightarrow 2\gamma \cosh \phi - 1 - \gamma^2 = 0$$

(23)

According to the relationship between Eqs. (23) and (22), the searched macroscopic criterion has the following form when $\Sigma_{eq} = 0$:

$$\Gamma \cosh \left[ A \ln \left( 1 - 3\alpha \frac{\Sigma_m}{\sigma_0} \right) \right] - 1 - \Gamma^2 = 0$$

(24)

where $\Gamma = [a^2W_0(f_p) + 2a^2W_0(f_p)]^A$.

In order to determine the parameter $A$, the following conditions should be verified when $\alpha \to 0$, both for tension and compression:

$$\lim_{\alpha \to 0} \cosh \left[ A \ln \left( 1 - 3\alpha \frac{\Sigma_m}{\sigma_0} \right) \right] = \cosh \left( \frac{3}{2} \frac{\Sigma_m}{\sigma_0} \right), \quad \lim_{\alpha \to 0} \Gamma = f$$

(25)

Finally, an expression of parameter $A$ is obtained:

$$A = \text{sign}(\Sigma_m) \frac{1 - [1 - a^2W_0(f_p) + 2a^2W_0(f_p)]/9}{2\alpha}$$

(26)
The parameter $A$ depends not only on the compressible parameter $a$ and porosity $f$, but also on the sign of $\Sigma_m$.

Compared with the Gurson’s criterion, the deviatoric part $\Sigma_{eq}/\sigma_0$ should be added in Eq. (24). This will be considered in the next special case of $f \to 0$.

(3) Case of the porosity $f \to 0$

When the void volume fraction of the porous material becomes zero ($f \to 0$), the searched macroscopic criterion should reduce to the Mises–Schleicher criterion (Eq. (19)):

$$\frac{\Sigma_{eq}^2}{\sigma_0^2} + 3a \frac{\Sigma_m}{\sigma_0} = 1 = 0$$

(27)

When $f \to 0$, the limitation of the left hand side of Eq. (24) is

$$\lim_{f \to 0} \left\{ 2f \cosh \left[ A \ln \left( 1 - 3a \frac{\Sigma_m}{\sigma_0} \right) \right] - 1 - f^2 \right\} = -1$$

(28)

With this property, the expression of the searched macroscopic criterion is proposed as follows:

$$\frac{\Sigma_{eq}^2}{\sigma_0^2} + 2B \left[ A \ln \left( 1 - 3a \frac{\Sigma_m}{\sigma_0} \right) \right] - 1 - f^2 = 0$$

(29)

The parameters $B$ and $C$ were introduced to consider the effects of porosity $f$ and the compressibility of the matrix $a$ on the deviatoric stress $\Sigma_{eq}$. When $f \to 0$, we should find $\lim_{f \to 0} B = 1$ and $\lim_{f \to 0} C = 1$.

(4) Case of $\Sigma_m = 0$ for the determination of the parameters $B$ and $C$.

The expressions of the parameters $B$ and $C$ are determined by the property of continuity between tension and compression. Unlike the case of purely hydrostatic loadings, we do not have the exact solution under the deviatoric loading which is difficult to be obtained analytically. To this end, the result of $\Sigma_{eq}/\sigma_0 = 1 - f$ proposed by Gurson’s criterion will be adopted here as an approximation. The expression of $\Sigma_{eq}/\sigma_0$ obtained from the macroscopic criterion (Eq. (29)) when $\Sigma_m = 0$ is

$$\frac{\Sigma_{eq}^2}{\sigma_0^2} = B (1 - f)^2$$

(30)

The above special cases (1, 2 and 3) should be considered at the same time during the determination of the parameters $B$ and $C$:

$$\lim_{f \to 0} \left[ B - 3a C \frac{\Sigma_m}{\sigma_0} \right] = 1, \quad \lim_{f \to 0} B = 1, \quad \lim_{f \to 0} C = 1$$

(31)

With Eqs. (30) and (31), considering the continuity between tension and compression, the expressions of the parameters $B$ and $C$ can be obtained:

$$B = \frac{(1 - f)^2}{(1 - f)^2}, \quad C = 1 - f$$

(32)

Taking into account these special conditions and requirements, the searched macroscopic yield function of a porous medium with a Mises–Schleicher matrix can be proposed as follows:

$$\phi = \frac{\Sigma_{eq}^2}{\sigma_0^2} + 2f \cosh \left[ A \ln \left( 1 - 3a \frac{\Sigma_m}{\sigma_0} \right) \right] - 1 - f^2 = 0$$

(33)

Compared with the Gurson’s criterion (Eq. (2)), the macroscopic criterion (Eq. (33)) has a similar expression. It is worth to notice that the criterion (Eq. (33)) is analogous to the heuristic one used in Nahshon and Hutchinson (2008) (see also Tvergaard, 1981; Tvergaard and Needleman, 1984). The damage parameter $f$ was introduced, which replaces the porosity $f$ in Gurson’s criterion and plays an important role in the macroscopic behaviors (asymmetry between tension and compressive loadings). $f$ is a function of the plastic compressible parameter $a$, the porosity $f$, and also the type of loading, sign ($\Sigma_m$).

The proposed macroscopic criterion (Eq. (33)) was assessed and validated by comparing it with numerical results (upper and lower bounds) by Pastor et al. (2013) for strong and weak plastic compressibility of the Mises–Schleicher type matrix and for different porosities. This criterion (Eq. (33)) improves the existing ones proposed by Lee and Oung (2000) and Durban et al. (2010), respectively.

### 2.4. Porous medium with a Drucker–Prager type matrix

Drucker–Prager criterion is widely used in geomaterials to capture the plastic compressibility and the asymmetry between tension and compression. For a porous material with a Drucker–Prager type matrix, the matrix in Fig. 1 obeys the following macroscopic criterion for a hollow sphere with a Drucker–Prager criterion (Eq. (32)).

$$\phi^m = \sigma_{eq} + 3a \sigma_m - \sigma_0 \leq 0$$

(34)

Jeong (2002) proposed a heuristic macroscopic criterion by considering some special cases (hydrostatic loadings, the porosity $f \to 0$, and the compressible parameter $a \to 0$):

$$\left\{ 2f \cosh \left[ A \ln \left( 1 - 3a \frac{\Sigma_m}{\sigma_0} \right) \right] - 1 - f^2 \right\} = 0$$

(35)

Recently, Guo et al. (2008) obtained an implicit expression of the macroscopic criterion for a hollow sphere with a Drucker–Prager type matrix by means of limit analysis techniques.

In order to obtain a closed-form expression, the implicit criterion was approximated by

$$\left\{ \frac{\Sigma_{eq}}{\sigma_0} \right\}^2 + 2f \cosh \left[ \gamma^{-1} \ln \left( 1 - 3a \frac{\Sigma_m}{\sigma_0} \right) \right] - 1 - f^2 = 0$$

(36)

where

$$\gamma = \frac{2a}{2a + \text{sign}(\Sigma_m)} \quad s = 1 + 2a \text{sign}(\Sigma_m)$$

On the other hand, a porous material with a Drucker–Prager matrix was studied in Magbous et al. (2009) by the modified secant modulus procedure. The Drucker–Prager criterion reads...
\[ \phi^m = \sigma_d + T(\sigma_m - h) \leq 0 \]  
(37)

where \( \sigma_d = \sqrt{\sigma' : \sigma} \) is the equivalent stress (with \( \sigma' = \sigma - \sigma_m \)), \( h \) represents the hydrostatic tensile strength, and \( T \) denotes the frictional coefficient.

By using the modified secant method, interpreted by Suquet (1995) (see also Ponte Castaneda and Suquet, 1998) as equivalent to the variational method of Ponte Castaneda (1991), Maghous et al. (2009) succeeded in deriving a macroscopic criterion in the following form:

\[
F(\sigma, f, T) = \frac{1 + 2f/3}{T^2} \sigma_d^2 + \left( \frac{3f}{2T^2} - 1 \right) \sigma_m^2 + 2(1-f)h\sigma_m - (1-f)^2h^2 = 0
\]  
(38)

The representation of this elliptic criterion corresponds to a closed surface. It presents a tension-compression asymmetry which is a manifestation of the Drucker–Prager type matrix. Note that the criterion (Eq. (4)) established by Ponte Castaneda (1991) is retrieved by Eq. (38) in the limit of a von Mises solid phase.

3. Macroscopic criteria of ductile porous materials with two populations of voids

Porous materials with one population of pores have been studied in Section 2. In reality, different sizes of pores can be found in porous geomaterials, e.g., chalk, concrete, and argillite. Many works focus on this class of porous materials (see for instance Talukdar et al., 2004; Vincent et al., 2008, 2009, 2014a, b; Shen, 2011; Alam et al., 2010; Cariou et al., 2013; Shen and Shao, 2016a, b). To this end, these double porous materials will be investigated in this section. Explicit macroscopic criteria will be established which take into account different influences of small and large pores on the macroscopic mechanical behavior.

For this purpose, the schematization of the studied double porous material containing two populations of spherical voids is illustrated in Fig. 2. The voids in these two populations are assumed to be spherical and isotropically distributed. Different scales will be considered: macroscale, mesoscale and microscale. The separation between the two scales of the voids allows performing a two-step homogenization.

In Fig. 2, we denote \(|\Omega|\) the total volume of the RVE, \(\Omega_1\) the domain occupied by the voids at the microscale (the smallest scale), and \(\Omega_2\) the one of the voids at the mesoscale (the intermediate scale). With these notations, the micro-porosity \( f \) and the meso-porosity \( \phi \) can be expressed as

\[
f = \frac{|\Omega_1|}{|\Omega - \Omega_2|}, \quad \phi = \frac{|\Omega_2|}{|\Omega|}
\]  
(39)

3.1. Double porous media with a Drucker–Prager solid phase at the macroscale

For various porous geomaterials, polymers or other pressure-sensitive materials, plastic compressibility is one of the salient features that must be taken into account. To this end, the solid phase at the microscale is assumed to obey a Drucker–Prager type criterion (Eq. (37)). In order to consider the effects of the micro-porosity \( f \) and the meso-porosity \( \phi \) on the effective behavior of double porous material and to derive a closed-form expression of the macroscopic strength criterion, a two-step homogenization procedure (from microscale to mesoscale and from mesoscale to macroscale) will be adopted.

Concerning the first homogenization from microscale to mesoscale (see Fig. 2b), the criterion (Eq. (38)) obtained by Maghous et al. (2009) for one population of pores is adopted directly. The influence of micro-porosity \( f \) and compressibility in the solid phase at the microscale is considered in Eq. (38). Furthermore, this criterion has an elliptic expression which is similar to Eq. (6) used in Section 2.2. Taking advantage of this property, the result (Eq. (18)) in Section 2.2 obtained by means of limit analysis approach can be used to describe the second homogenization from mesoscale to macroscale (see Fig. 2a). The meso-porosity \( \phi \) is taken into account in this step. Finally, the macroscopic criterion for a double porous material with a Drucker–Prager solid phase at the microscale is given by

\[
\beta \frac{\Sigma^2}{\sigma_0^2 + L^2/(18\alpha)} + \frac{9\alpha}{2} \left[ \frac{\Sigma_m - L(1 - \phi)/9\alpha}{\sqrt{\sigma_0^2 + L^2/(18\alpha)}} \right]^2
\]

\[
+ 2\phi \cosh \left[ \frac{3\sqrt{\beta}}{2} \frac{\Sigma_m}{\sqrt{\sigma_0^2 + L^2/(18\alpha)}} \right] - 1 - \phi^2 = 0
\]  
(40)

where the parameters \( \beta, \alpha, L \) and \( \sigma_0 \) are the functions of micro-porosity \( f \) and the properties, \( T \) and \( h \), of the solid phase:

\[
\beta = \frac{2}{3} \left( \frac{1 + 2f/3}{T^2} \right), \quad \frac{9\alpha}{2} = \frac{3f}{2T^2} - 1, \quad L = -2(1-f)h, \quad \sigma_0 = (1-f)h
\]  
(41)

The macroscopic criterion for a double porous medium having a von Mises solid phase at the microscale can be retrieved from Eqs. (40) and (41) by considering the following properties: \( T \to 0 \), \( h \to \infty \) and \( T h = \sqrt{2/3}\sigma_0 \).

In many engineering structures, the pore pressure due to the saturation has a great influence on the macroscopic behaviors of porous geomaterial. To consider this effect, Eqs. (40) and (41) were generalized in Shen et al. (2014a) for the double porous material saturated by two different pore pressures at the microscale and mesoscale. The effective stress problems are also discussed. By considering Eshelby-like trial velocity fields in the limit analysis approach for the transition from mesoscale to macroscale, a further improvement has been made in Shen et al. (2014b).

3.2. Macroscopic criterion of double porous material obtained by the modified secant method

The kinematical limit analysis approach has been adopted in Section 3.1 in order to account for the effect of porosity \( \phi \) at the mesoscale. In this section, an alternative method will be used for this second homogenization procedure to establish a macroscopic...
criterion of double porous material with a Drucker–Prager type matrix at the microscale.

For the first homogenization shown in Fig. 2b, the criterion (Eq. (38)) proposed by Maghous et al. (2009) will be adopted for the effects of micro-porosity f and the compressibility of the solid phase. Based on this criterion and an associated flow rule, one obtains the plastic strain rate of the porous matrix (Fig. 2b):

\[
d = \frac{T^2}{1 + 2\phi/3} \frac{d_0}{6d} \left[ 1 + \frac{2f/3}{T^2} \sigma' + \left( \frac{3}{2T^2} - 1 \right) \frac{2\sigma_m}{3} \mathbf{1} \right]
\]  

(42)

where \(d_0 = \sqrt{\mathbf{d} : \mathbf{d}}\), and \(\mathbf{d}' = \mathbf{d} - \mathbf{d}_\text{m} 1\) denotes the deviatoric part of the plastic strain rate tensor \(\mathbf{d}\).

As the elastic domain delimited by the yield function \(F(\sigma, f, T)\) is convex and closed in the stress space, the support function \(\pi^{\text{mp}}\) can be readily expressed as

\[
\pi^{\text{mp}} = (1 - f)h \sqrt{\frac{3f}{3f - 2T^2}} \frac{T^2}{1 + 2\phi/3} \frac{d_0}{6d} \left[ d^2 + \frac{1 + 2f/3}{3f/2 - T^2} d^2 \right] - (1 - f)h^2 \mathbf{d} \frac{2T^2}{3f - 2T^2} \frac{d_0}{6d} 
\]  

(43)

where \(d_0 = \text{tr} \mathbf{d}\) is the volumetric deformation in the porous matrix.

The stress–strain relationship derived from the support function \(\pi^{\text{mp}}\) can be put in the following form which requires the secant bulk and shear moduli and introduces a hydrostatic prestress \(\sigma^p\):  

\[
\sigma^{\text{mp}} = \frac{\partial \pi^{\text{mp}}}{\partial \mathbf{d}} = 2\mu^{\text{mp}} \mathbf{d}' + \kappa^{\text{mp}} \mathbf{d}_\text{m} 1 + \sigma^p \mathbf{1}
\]  

(44)

Note that the secant moduli \(\mu^{\text{mp}}\) and \(\kappa^{\text{mp}}\) in Eq. (44) are non-uniform because they depend on the non-uniform strain rate tensor \(\mathbf{d}\). As in Maghous et al. (2009), the effective strain rate \(\mathbf{d}^{\text{eff}}\) will be considered, which is an appropriate average of \(\mathbf{d}\) over the porous matrix to capture the effect of loading history on the nonlinear plastic properties. Therefore, the effective moduli \(\mu^{\text{mp}}\) and \(\kappa^{\text{mp}}\) depend on the macroscopic strain rate \(\mathbf{D}\) of the double porous matrix. The effective volumetric and deviatoric strain rates of the porous matrix are defined as

\[
\mathbf{d}^{\text{eff}} = \sqrt{<\mathbf{d}^{\text{eff}} : \mathbf{d}^{\text{eff}}>} \quad \text{and} \quad \mathbf{d}^{\text{eff}} = \sqrt{<\mathbf{d}^{\text{eff}} : \mathbf{d}^{\text{eff}}>} \mathbf{1}
\]  

(45)

from which the state equation can be rewritten as

\[
\sigma^{\text{mp}} = \mu^{\text{mp}} \left( \mathbf{d}^{\text{eff}}, \mathbf{d}^{\text{eff}} \right) : \mathbf{D} + \sigma^p \mathbf{1}
\]  

(46)

The homogenized stress–strain relations of the double porous medium can be expressed in the following form:

\[
\mathbf{D} = \mathbf{C}^{\text{hom}} : \mathbf{D} + \mathbf{2} \sigma^p \mathbf{1}
\]  

(47)

where \(\Sigma\) is the stress tensor at the mesoscale in the double porous matrix, and \(\Sigma^p\) denotes the mesoscopic prestress which can be deduced from the Levin's theorem, \(\Sigma^p = (\kappa^{\text{hom}}/\kappa^{\text{mp}}) \sigma^p_{\text{eq}}\).

The thermodynamic potential of the double porous medium reads:

\[
W = \frac{1}{2} \mathbf{D} : \mathbf{C}^{\text{hom}} : \mathbf{D} + \mathbf{2} \sigma^p \text{tr} \mathbf{D}
\]  

(48)

The corresponding state equations are easily deduced as (see also Eq. (47)):

\[
\Sigma^{\text{m}} = \kappa^{\text{hom}} \left( \mathbf{D}_v + \frac{\sigma^p_{\text{eq}}}{\kappa^{\text{eq}}} \right), \quad \Sigma^{\text{d}} = 2 \mu^{\text{hom}} D_d
\]  

(49)

Following Dormieux et al. (2002), the effective strain rates associated with the double porous medium are related to \(\mathbf{D}\) and given by

\[
\frac{1}{2} (1 - \phi) \left( \mathbf{d}^{\text{eff}} \right)^2 = \frac{1}{2} \frac{\partial \pi^{\text{hom}}}{\partial \mathbf{d}} \left( \mathbf{D}_v + \frac{\sigma^p_{\text{eq}}}{\kappa^{\text{eq}}} \right)^2 - \frac{\kappa^{\text{hom}}}{\kappa^{\text{mp}}} \left( \frac{\sigma^p_{\text{eq}}}{\kappa^{\text{eq}}} \right)^2 \mathbf{D}_v + \frac{\kappa^{\text{hom}}}{\kappa^{\text{eq}}} \mathbf{D}_d
\]  

\[
\frac{1}{2} (1 - \phi) \left( \mathbf{d}^{\text{eff}} \right)^2 = \frac{1}{2} \frac{\partial \pi^{\text{hom}}}{\partial \mathbf{d}} \left( \mathbf{D}_v + \frac{\sigma^p_{\text{eq}}}{\kappa^{\text{eq}}} \right)^2 - \frac{\kappa^{\text{hom}}}{\kappa^{\text{mp}}} \left( \frac{\sigma^p_{\text{eq}}}{\kappa^{\text{eq}}} \right)^2 \mathbf{D}_v + \frac{\kappa^{\text{hom}}}{\kappa^{\text{eq}}} \mathbf{D}_d
\]  

(50)

The homogenized secant moduli \(\kappa^{\text{hom}}\) and \(\mu^{\text{hom}}\) are evaluated with the help of the Hashin–Shtrikman upper bounds:

\[
\kappa^{\text{hom}} = \frac{\mu^{\text{hom}}}{\kappa^{\text{eq}}} \left( 4(1 - \phi) \mu^{\text{mp}} / \kappa^{\text{mp}} + \frac{\phi \mu^{\text{mp}}}{\kappa^{\text{eq}}} \right)
\]  

(51)

\[
\mu^{\text{hom}} = \mu^{\text{mp}} \frac{(1 - \phi) \left( 9 \sigma^p_{\text{eq}} + 8 \sigma^{\text{mp}}_{\text{eq}} \right)}{\kappa^{\text{eq}} (9 + 6\phi) + \mu^{\text{mp}} (8 + 12\phi)}
\]  

(52)

Combining Eqs. (44), (49)–(51), the macroscopic yield criterion of the double porous material with two populations of spherical cavities at two different scales takes the following form:

\[
\Phi(\Sigma, \phi, f, T) = \Theta \Sigma^2 + \Gamma \Sigma^m
\]  

\[
+ 2(1 - f)h(1 - \phi) \Sigma - (1 - f)^2 (1 - \phi)^2 h^2 = 0
\]  

(52)

where

\[
\Theta = \frac{1 + 2f/3}{T^2} \left( 6T^2 - 13f/3 - 12f/3 \phi + 9 \right)
\]  

\[
\Gamma = \frac{3 + 2 + f}{T^2} \phi + \frac{3f}{2T^2} - 1
\]  

This closed-form plastic yield function explicitly depends on the two porosities (\(f\) and \(\phi\)) at two different scales.
4. Applications

Some recently established macroscopic strength criteria have been presented in Sections 2 and 3 for different types of porous geomaterials having one or two populations of pores. The effects of porosities at different scales and the influence of the matrix compressibility on the macroscopic mechanical behaviors are explicitly taken into account. In this section, we will provide some applications of these strength criteria to describe the overall responses of porous geomaterials.

4.1. Application of the macroscopic criterion (Eq. (33)) to sandstone

As a first application, a typical porous geomaterial “Vosges sandstone” will be considered. This material comes from the Vosges mountains in France, which has been studied in a series of experimental investigations (Khazraei, 1996; Shao and Khazraei, 1996; Besuelle et al., 2000). The Vosges sandstone is mainly composed of quartz grains (93%), with a few percent of feldspar and white mica. The porosity is about 22%. This sandstone is considered to be a typical porous quasi-brittle rock. With the increase of confining pressure, its mechanical behavior exhibits a brittle–ductile transition from low to high confining pressures (Menéndez et al., 1996; Besuelle et al., 2000).

In order to describe the macroscopic behavior of the Vosges sandstone taking into account the effects of porosity and the asymmetric between tension and compression, the macroscopic criterion (Eq. (33)) established in Section 2.3 will be applied. We aim now at formulating and implementing a complete constitutive plastic model. For most porous geomaterials, a significant plastic hardening can be observed in experimental phenomena. In this application, the plastic hardening of the solid phase is considered via the evolution of \( \sigma_0 \) used in Eq. (33), as a function of the equivalent plastic strain \( \varepsilon_{eq}^{p} \) in the solid phase. An isotropic hardening is considered in the following form for the sandstone:

\[
\sigma = \sigma_0 + \left[H\left(\varepsilon_{eq}^{p}\right)\right]^{m}
\]  

(53)

With this plastic hardening parameter, the macroscopic criterion (Eq. (33)) becomes

\[
\Phi(\Sigma, f, \alpha, \varpi) = \frac{\Sigma^2}{2\varepsilon_{eq}^{p}} - 3\alpha \frac{1-f}{1+f} \frac{\varepsilon_{eq}^{p}}{\varpi} \\
+ 2\lambda \cosh \left[A \ln \left(1 - 3\alpha \frac{\Sigma_{m}}{\varpi}\right)\right] - 1 - I^2 = 0
\]

(54)

Adopting the normality rule, the plastic flow is given by

\[
D^p = \lambda \frac{\partial \Phi}{\partial \Sigma} : \Sigma + \frac{\partial \Phi}{\partial f} : f + \frac{\partial \Phi}{\partial \varpi} : \varpi = 0
\]

(55)

where \( D^p \) denotes the macroscopic plastic strain rate. The plastic multiplier \( \lambda \) can be determined by the plastic consistency condition:

\[
\Phi(\Sigma, f, \alpha, \varpi) = \frac{\partial \Phi}{\partial \Sigma} : \Sigma + \frac{\partial \Phi}{\partial f} : f + \frac{\partial \Phi}{\partial \varpi} : \varpi = 0
\]

(56)

The tangent elastoplastic operator of the studied porous material can be determined as follows:

\[
\mathbb{E} = \left\{ \begin{array}{ll}
\text{C} & \left( \Phi \leq 0, \; \Phi \leq 0 \right) \\
\text{C} - \left( \text{C} : \frac{\partial \Phi}{\partial \Sigma} \right) \left( \frac{\partial \Phi}{\partial \Sigma} : \text{C} \right) & \left( \Phi = 0, \; \Phi > 0 \right)
\end{array} \right.
\]

(61)

where \( \text{C} \) is the macroscopic elastic modulus tensor of the porous material. By adopting a Mori-Tanaka homogenization scheme, the macroscopic elastic properties (\( \kappa, \mu \)) are related to those of the matrix (\( \kappa_s, \mu_s \)) by

\[
\kappa = \frac{4(1-f)\kappa_s \mu_s}{4\mu_s + 3\kappa_s}, \quad \mu = 1 + 6f(\kappa_s + 2\mu_s)/(9\kappa_s + 8\mu_s)
\]

(62)

The macroscopic elastic properties depend on the evolution of porosity during loading. Then the proposed micromechanical constitutive model is implemented in a finite element code (ABAQUSS) via a subroutine UMAT. For the purpose of applying to sandstone porous geomaterial, all the model parameters should be identified. According to the experimental data, the macroscopic elastic properties (\( \kappa, \mu \)) can be calculated from the elastic regime of the porous sandstone. With the relationship of Eq. (62) and the porosity \( f = 0.22 \), the compressible modulus \( \kappa_s \) and the shear one \( \mu_s \) of the solid matrix can be calculated. The corresponding Young’s modulus \( E_s = 30 \) GPa and the Poisson’s ratio \( v_s = 0.3 \). The plastic compressible parameter \( \alpha \) and the initial yield stress \( \sigma_0 \) are identified from a triaxial compression test. All calibrated parameters are given in Table 1.
Using the same set of values given in Table 1, comparisons between experimental data and numerical simulations with different confining pressures (10 MPa and 40 MPa) are illustrated in Figs. 3 and 4. Good agreements can be found for both low and high confining pressures. The axial and lateral strains are well predicted by the proposed associated model which is able to capture the main mechanical features of the porous sandstone.

4.2. Application of the macroscopic criterion (Eq. (40)) to chalk

In the second application, a complete constitutive model will be established for a double porous material and applied to describing the mechanical behavior of the porous “Lixhe chalk” which is from the Upper Campanian age and drilled in the CBR quarry near Liége (Belgium). This material has been studied in a series of previous experimental investigations because its mechanical behavior is qualitatively close to that of North Sea reservoir chalks (Schroeder, 2003). It is composed of more than 98% of CaCO₃, less than 0.8% of SiO₂ and 0.15% of Al₂O₃. The porosity of Lixhe chalk ranges from 42% to 44%, which is a highly porous rock. According to Talukdar et al. (2004) and Alam et al. (2010), two kinds of porosity are observed in chalk. The large porosity φ at the mesoscale is about 5% and the small f is about 40% in the matrix. The macroscopic plastic yield function (Eq. (40)) presented in Section 3.1 will be applied to describing the elastoplastic behavior of this porous chalk.

The plastic hardening of the solid phase at the microscale is taken into account via the evolution of the frictional coefficient T as a function of the equivalent plastic strain:

\[ T = T_0 \left[ 1 + b \left( \frac{\sigma_{eq}}{\sigma_0} \right)^m + c \left( e^{\sigma_{eq}} - 1 \right) \right] \]  

(63)

where \( b, m, c \) and \( n \) are the parameters of the hardening law, which will be determined by simulating a hydrostatic compression test.

In order to better describe the plastic volumetric deformation, a non-associated plastic flow rule is proposed. Inspired by the work of Maghous et al. (2009) and Shen et al. (2012a, 2013a), the following function is adopted as the macroscopic plastic potential by taking a similar form as the yield function:

\[ G = \left( \frac{2}{3} + 4f \right) \left( \frac{\Sigma_{eq}}{\Sigma_0} \right)^2 + \left( \frac{3f}{2} - Tr \right) \left( \left[ \Sigma_m + \frac{2(1 - f)hT^2}{3f - 2Tr} (1 - \phi) \right] / \Sigma_0 \right)^2 + 2\phi \cosh \left( \frac{3f}{2} + f \Sigma_m / \Sigma_0 \right) \]

\[ \Sigma_0 = (1 - f)hT \sqrt{3f - 2Tr} \]

where the parameter \( t \) is the dilatancy coefficient which controls the transition between volumetric contractance and dilatancy under deviatoric loading. As the rate of volumetric dilatancy generally varies with plastic deformation history, for simplicity, the same form as Eq. (63) is adopted for \( t \):

\[ t = T_0 \left[ 1 + b \left( \frac{\sigma_{eq}}{\sigma_0} \right)^m + c \left( e^{\sigma_{eq}} - 1 \right) \right] \]  

(65)

According to the macroscopic potential (Eq. (64)), the plastic flow rule is given by

\[ D^p = \lambda \frac{\partial G}{\partial \Sigma} (\Sigma, f, \phi, T, t) \]  

(66)

The equivalent plastic strain of the solid phase is given by

\[ \varepsilon_{eq}^p = \frac{\Sigma : D^p}{(1 - f)(1 - \phi) \left[ Th + \frac{(r - T)\Sigma_m}{(T - T_0)(1 - \phi)} \right]} \]  

(67)
The evolutions of micro-porosity \( f \) and meso-porosity \( \phi \) are determined as

\[
\dot{f} = (1-f)\left\{ \frac{\dot{\Omega}_m + \dot{\bar{\varepsilon}_1}}{\dot{\varepsilon}_m + \dot{\bar{\varepsilon}_1}} - (1-f)\frac{\dot{\Omega}_m}{\dot{\varepsilon}_m + \dot{\bar{\varepsilon}_1}} \right\} \\
\dot{\phi} = (1-\phi)\left\{ \frac{\dot{\Omega}_m + \dot{\bar{\varepsilon}_2}}{\dot{\varepsilon}_m + \dot{\bar{\varepsilon}_2}} - (1-\phi)\frac{\dot{\Omega}_m}{\dot{\varepsilon}_m + \dot{\bar{\varepsilon}_2}} \right\}
\]

(68)

Then the proposed micromechanical constitutive model is implemented in a finite element code (ABAQUS) via a subroutine UMAT. Before it is applied to describing the macroscopic behavior of the Lixhe chalk, we need to determine the elastic and plastic parameters of the proposed non-associated model.

The plastic parameters are determined by simulating the hydrostatic compression test (see Fig. 5). A good agreement is found between the numerical simulation and experimental data. The chalk behavior is significantly sensitive to the confining pressure. With the same values given in Table 2, numerical simulations are now performed for triaxial compression tests with low and high confining pressures in order to verify the capacity of the proposed model.

Fig. 6 shows the comparison between experimental data and numerical results for low confining pressure of 4 MPa. The material fails with a small value of the axial strain. The cases of high confining pressures (14 MPa and 17 MPa) are shown in Figs. 7 and 8, respectively. One can see a good agreement between the numerical simulation and experimental data. The proposed non-associated model is able to capture the main aspects of the mechanical behaviors of Lixhe chalk by considering the effects of microstructure.

Figs. 9 and 10 show the evolutions of micro-porosity \( f \) in the matrix and of meso-porosity \( \phi \) as functions of axial strain during triaxial compression tests with different confining pressures (4 MPa, 14 MPa and 17 MPa). We can see that the evolutions of \( f \) and \( \phi \) are different. These changes of the microstructure affect the macroscopic plastic criterion (Eq. (40)). The macroscopic mechanical behavior of the Lixhe chalk is clearly sensitive to these two types of porosities.

### 4.3. Application of the macroscopic criterion (Eq. (52)) to COx argillite with the effects of inclusions

In the above sections, the effects of porosity have been taken into account on the macroscopic behavior of porous geomaterial. For many materials used in engineering structure, for instance concrete and rocks, complex microstructure with mineral inclusions, granular aggregates and pores can be found. Taking the COx argillite for example, the effects of pores at different scales and the influences of inclusions will be both considered.

Because of its low permeability and high mechanical strength, the COx argillite is considered as a potential geological barrier. According to Andra (2005) and Robinet (2008), the COx claystone is composed of 40%–50% of clay minerals, 20%–27% of calcite grains, 23%–25% of quartz grains and 5%–10% of minor minerals. The porosity of COx claystone varies from 11.04% to 13.84%. The average pore size (tens of nm) is significantly smaller than that of calcite and quartz grains. The majority of pores are located inside the clay matrix. At the microscale, the porous clay matrix can be treated as an assembly of clay particles and inter-particular pores. Furthermore, the clay particle contains also small voids (intra-particle pores) between clay sheets. In order to describe elastoplastic and damage behaviors, some phenomenological models (for instance Chiarelli et al., 2003; Shao et al., 2006; Hoxha et al., 2007) and micromechanical constitutive models based on different homogenization techniques (Abou-Chakra Guéry et al., 2008; Shen et al., 2012a, 2013a; Shen and Shao, 2015b) have been proposed for COx argillite. In this study, the effects of two populations in the clay matrix at the microscale and the ones of minerals grains (calcite and quartz) at the mesoscale will be taken into account. The RVE of the studied heterogeneous material is schematized in Fig. 11.

---

**Table 2**

Typical values of parameters for the non-associated model.

<table>
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<tr>
<th>( E_s ) (GPa)</th>
<th>( v_e )</th>
<th>( T_0 )</th>
<th>( t_0 )</th>
<th>( h )</th>
<th>( b )</th>
<th>( m )</th>
<th>( c )</th>
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<td>0.018</td>
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</table>

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**Fig. 5.** Simulation of a hydrostatic compression test on Lixhe chalk.

**Fig. 6.** Simulation of a triaxial compression test on Lixhe chalk with 4 MPa confining pressure.

**Fig. 7.** Simulation of a triaxial compression test on Lixhe chalk with 14 MPa confining pressure.
We denote $U$ as the total volume of RVE (Fig. 11), $U_s$ the domain occupied by the intra-particle pores, and $U_b$ the one occupied by the inter-particle pores. $U_m$ is the volume of the solid phase in particles. At the mesoscopic scale, $U_{I1}$ and $U_{I2}$ denote the volumetric domains of mineral inclusion phase 1 and inclusion phase 2, respectively. With these notations, the porosity $f$ at the intra-particular scale (Fig. 11c) and the one $\phi$ at the inter-particular scale (Fig. 11b) of the porous matrix, as well as the volumetric fractions of inclusions ($\rho_1, \rho_2$) can be calculated:

$$
\rho_1 = \frac{U_{I1}}{U}, \quad \rho_2 = \frac{U_{I2}}{U} \\
\phi = \frac{U_b}{U_m + U_s + U_b} \\
f = \frac{U_s}{U_m + U_s}
$$

(69)

As shown in Fig. 11a, the COx argillite is made of a porous clay matrix and mineral grains of calcite and quartz at the mesoscale. The criterion (Eq. (52)) obtained in Section 3.2 will be applied to describing the effective behaviors of the clay matrix considering the two populations of pores (Fig. 11b and c) and the compressibility. The influences of calcite and quartz grains on the macroscopic plastic behavior will be estimated by using an incremental approach proposed by Hill (1965). The advantage of this approach is the possibility to consider several families of mineral inclusions and also complex loading path due to its incremental form.

4.3.1. Principle of incremental approach

In the incremental approach, the local stress rate $\dot{\sigma}$ of each constituent phase is related to the local strain rate $\dot{\varepsilon}$ by the local tangent stiffness operator $L(z)$:

$$
\dot{\sigma}(z) = L(z) : \dot{\varepsilon}(z)
$$

(70)

At each iteration step, it is possible to introduce a tangent localization tensor $A(z)$ to relate the local strain rate $\dot{\varepsilon}(z)$ to the macroscopic strain rate $\dot{\varepsilon}$:

$$
\dot{\varepsilon}(z) = A(z) : \dot{\varepsilon}
$$

(71)

The rate form of the macroscopic constitutive relations can be written in the general form:

$$
\dot{\Sigma} = \dot{\varepsilon}^{\text{hom}} : \dot{E}, \quad \dot{\varepsilon}^{\text{hom}} = < \dot{\varepsilon}(z) : A(z) >
$$

(72)

where $\dot{\varepsilon}^{\text{hom}}$ denotes the effective tangent stiffness operator which is obtained as the average over the RVE of the product of the local tangent stiffness and tangent strain localization tensor on all constituent phases.

In general, the local tangent stiffness is not uniform in each phase due to the material heterogeneity. As a consequence, it is not possible to provide a closed-form expression of its average. Some simplifications are then needed to make the homogenization procedure computationally operable. For this purpose, at any point, $z,$
of a given phase \( r \), the local constitutive relation is approximated by

\[
\mathbf{Y} \mathbf{E}(r), \quad \mathbf{\sigma}(\varepsilon) = \mathbf{A}_r \mathbf{\varepsilon}
\]

(73)

The local tangent operator \( \mathbf{A}_r \) is evaluated at a suitable reference strain state, which is classically taken as the average value of local strain field in the phase \( r \). This implies that the tangent stiffness is uniform in each phase. Accordingly, the incremental form of the strain localization relation is simplified as follows:

\[
\mathbf{\dot{\varepsilon}} = \mathbf{A}_r \mathbf{\dot{\varepsilon}}
\]

(74)

where \( \mathbf{A}_r \) is the tangent localization operator corresponding to the average strain per phase \( r \).

With the matrix-inclusion morphology at the mesoscale, the Mori-Tanaka scheme (Mori and Tanaka, 1973) is considered for the evaluation of the tangent strain localization operator:

\[
\mathbf{A}_r = \left[ \mathbf{I} + \mathbf{p}^0_r : (\mathbf{\varepsilon}_r - \mathbf{0}) \right]^{-1} \left( \sum_{i=0}^{N} f_i \mathbf{A}_i^0 \right) \left[ \mathbf{I} + \mathbf{p}^0_r : (\mathbf{\varepsilon}_r - \mathbf{0}) \right]^{-1}
\]

(75)

where \( \mathbf{I} \) denotes the fourth-order unit tensor, \( \mathbf{p}_r^0 \) is the volumetric fraction of the phase \( r \).

It is worth recalling that the Hill tensor \( \mathbf{p}_r^0 \) in Eq. (75) depends on the inclusion form \( \mathbf{I}_r \), its orientation as well as the local stiffness of the matrix phase in the linear comparison composite \( \mathbf{I}_0 \). As the calcite and quartz grains have the same morphology, one notices that \( \mathbf{p}_1^0 = \mathbf{p}_2^0 = \mathbf{p}_3^0, \mathbf{p}_3^0 \) is related to the Eshelby tensor \( \mathbf{S}_r \) through:

\[
\mathbf{p}_3^0 = \mathbf{S}_r(\mathbf{I}_0) : \mathbf{I}_0^{-1}
\]

(76)

As the tangent stiffness depends on the direction of plastic deformation, the local tangent operator \( \mathbf{A}_r \) is inherently anisotropic in nature. This implies that in the general case Hill tensor \( \mathbf{p}_r^0 \) can be estimated only numerically. Once the Hill tensor and strain localization operator are calculated, the macroscopic tangent stiffness tensor can be easily deduced:

\[
\mathbf{\tilde{\varepsilon}}_{\text{hom}} = \sum_{r=0}^{N} f_r \mathbf{A}_r \quad (r = 0, 1, 2)
\]

(77)

where \( f_r \) is the volumetric fraction of the phase \( r \).

The incremental method has been applied to modeling nonlinear behaviors of various heterogeneous materials (see for example Doghri and Ouara, 2003; Chaboche et al., 2005; Abou-Chakra Guéry et al., 2008; Shen et al. 2012a), this method generally leads to a too stiff macroscopic response. Various techniques have been proposed to improve numerical predictions and to obtain softer macroscopic responses. Among those techniques, the isotropization technique has been introduced and largely used by various authors. The efficiency of this technique has also been confirmed for clayey rocks by Abou-Chakra Guéry et al. (2008), Shen et al. (2012a) and Chen et al. (2013). Therefore, for the present work, we adopt again the general scheme proposed by Bornert et al. (2001) to perform the isotropization of a fourth-order tensor.

The isotropic part of the tangent stiffness tensor of the clay matrix is given by

\[
\mathbf{\tilde{\varepsilon}}_{\text{iso}}^0 = (\mathbf{I} : \mathbf{\varepsilon}_0) \mathbf{I} \frac{1}{5} (\mathbf{K} : \mathbf{\varepsilon}_0) \mathbf{K} = 3\kappa_1 \mathbf{I} + 2\mu_4 \mathbf{K}
\]

(78)

Due to the isotropic character of \( \mathbf{\tilde{\varepsilon}}_{\text{iso}}^0 \) and the fact that the calcite and quartz grains are assumed to be of spherical form, a closed-form expression of the Eshelby tensor can be obtained:

\[
\mathbf{S}^E(\mathbf{\tilde{\varepsilon}}_{\text{iso}}^0) = \frac{3\kappa_1}{3\kappa_1 + 4\mu_4} \mathbf{I} + 3 \frac{6}{5} \frac{\kappa_1 + 2\mu_4}{3\kappa_1 + 4\mu_4} \mathbf{K}
\]

(79)

Accordingly, the Hill tensor \( \mathbf{p}_r^0 \) in Eq. (76) is replaced by

\[
\mathbf{p}_r^0 = \mathbf{S}^E(\mathbf{\tilde{\varepsilon}}_{\text{iso}}^0) : \mathbf{\varepsilon}_r^{-1} \quad (r = 1, 2)
\]

(80)

### 4.3.2. Numerical implementation

The proposed micro-macro model is implemented in a standard finite element code (ABAQUS) as a UMAT subroutine. We present here the numerical scheme for the local integration of the model at each Gauss point.

The loading path is divided into a limit number of steps. At the step \( n + 1 \), the material point at the macroscopic scale is subjected to a macroscopic strain \( \mathbf{E}_{n+1} = \mathbf{E}_n + \Delta \mathbf{E} \), while the strain at the step \( n \) is known and the strain increment \( \Delta \mathbf{E} \) is given. The problem to be solved here is to find the corresponding macroscopic stress state at the end of the loading step by using the incremental homogenization method presented above. The following numerical scheme is adopted:

1. Input data: \( \mathbf{E}_n, \Delta \mathbf{E} \): Phase 0 clay matrix: \( \epsilon_{0n}, \epsilon^0_0 \) and \( \epsilon^0_2 \); Phase 1 calcite grains: \( \epsilon_{1n}, \epsilon^0_1 \);
2. Phase 2 quartz grains: \( \epsilon_{2n} \).
3. Initially, the local strain increments in phases 1 and 2 are set equal to the macroscopic strain increment:

\[
\Delta \varepsilon^0_1 = \Delta \mathbf{E}, \quad \Delta \varepsilon^0_2 = \Delta \mathbf{E}
\]

(81)

4. In phases 1 and 2, the values of \( \Delta \varepsilon^0_1 \) and \( \Delta \varepsilon^0_2 \) are known, so one obtains \( \epsilon_{1n+1}, \epsilon_{2n+1} \) and \( \epsilon^0_1, \epsilon^0_2 \) which are the local stiffness tensors of phases 1 and 2, respectively.
5. The average local strain in the clay matrix is given by

\[
\Delta \varepsilon^0_0 = \frac{\Delta \mathbf{E} - \epsilon_{1n} \epsilon^0_1 - \epsilon_{2n} \epsilon^0_2}{1 - \epsilon_{1n} \epsilon^0_1 - \epsilon_{2n} \epsilon^0_2}
\]

(82)

6. At the iteration \( i \) for the phase 0, the values of \( \Delta \varepsilon^0_0, \epsilon_{0n}, \epsilon^0_0, \epsilon^0_2 \) are known, so one can compute \( \epsilon_{0n+1}, \epsilon^0_0, \epsilon^0_2 \) and \( \epsilon^0_0 \).
7. The Hill tensor is then numerically evaluated by Eq. (80).
8. The tensors \( \mathbf{A}^{0i}_1 \) and \( \mathbf{A}^{0i}_2 \) are given by

\[
\mathbf{A}^{0i}_1 = \left[ \mathbf{I} + \mathbf{p}^0_1 : (\mathbf{\varepsilon}^1_i - \mathbf{0}) \right]^{-1}
\]

(83)

\[
\mathbf{A}^{0i}_2 = \left[ \mathbf{I} + \mathbf{p}^0_2 : (\mathbf{\varepsilon}^2_i - \mathbf{0}) \right]^{-1}
\]

(84)
\[ A_0^i := \left( f_0 + f_1 A_0^i + f_2 A_0^i \right)^{-1} \]  
(85)

\[ G(\Sigma, \phi, f, t, t) = \Theta \Sigma \phi + \left( \frac{3}{2} + f + \frac{3f}{2\Theta} - 1 \right) \Sigma^2 + 2(1-f)h(1-\phi) \Sigma m \]  
\[ \Theta = 1 + 2f/3 \left( \frac{6(1-13f) - 6}{4f - 9(1-\phi + 1) \right) \]  
(94)

\[ A_1^i := A_0^i : A_0^i \]  
(86)

\[ A_2^i := A_0^i : A_0^i \]  
(87)

(9) Check the compatibility of local strains between two iterations for phases 1 and 2 and evaluate the error \( R \):

\[ R_1^i = A_1^i : \Delta E - \Delta e_1^i \]  
(88)

\[ R_2^i = A_2^i : \Delta E - \Delta e_2^i \]  
(89)

If \( \| R_1^i \| < \text{tolerance 1} \) and \( \| R_2^i \| < \text{tolerance 2} \), compatibility is reached. Otherwise, an additional iteration is needed until the convergence criterion is verified and one obtains

\[ \Delta e_1^{i+1} = \Delta e_1^i + R_1^i \]  
(90)

\[ \Delta e_2^{i+1} = \Delta e_2^i + R_2^i \]  
(91)

(10) The use of Mori-Tanaka scheme leads to the determination of macroscopic tangent stiffness tensor:

\[ \tilde{E}_{\text{hom}} := \left[ f_0 \tilde{E}_0 + f_1 \tilde{E}_1 : A_0^i + f_2 \tilde{E}_2 : A_0^i \right] : A_0^i \]  
(92)

so that the macroscopic stress tensor can be calculated as

\[ \Delta \Sigma_{n+1} = \tilde{E}_{\text{hom}} : \Delta E_{n+1} \]  
(93)

Like most geomaterials, the COx claystone exhibits a transition from volumetric compressibility to dilatancy during plastic deformation. In order to capture such a transition, it is generally necessary to use a non-associated plastic flow rule. Inspired by the previous work of Maghous et al. (2009) and based on the plastic criterion (Eq. (52)), we propose the following non-associated plastic potential for the double porous clay matrix:

\[ T = T_m - (T_m - T_0)e^{-b_1 \rho} \]  
(95)

\[ \tilde{T} = \tilde{T}_m - (\tilde{T}_m - \tilde{T}_0)e^{-b_2 \rho} \]  
(96)

where the dilatancy coefficient \( \tau \) defines the current plastic volumetric strain rate.

As for most porous materials, the porous clay matrix also exhibits a plastic hardening process, firstly due to the evolution of porosities. However, according to some previous studies (Abou-Chakra Guéry et al., 2008; Shen et al., 2012a), the current friction coefficient of the solid phase \( T \) and the dilatancy coefficient \( \tau \) vary with the plastic deformation. A complementary plastic hardening law is necessary for the solid phase of the clay matrix to consider the strength and the volumetric compressibility–dilatancy transition. Thus we consider here the frictional coefficient \( T \) and the dilatancy one \( \tau \) as functions of the equivalent plastic strain \( \rho \):

\[ \tilde{D}^p = \frac{\tilde{D}^p}{\delta \tilde{E}} (\Sigma, \phi, f, T, \tau) \]  
(97)

where \( \tilde{D}^p \) denotes the plastic strain rate in the clay matrix at the mesoscale.

The equivalent plastic strain of the solid phase in the clay matrix is given by

\[ \rho = \frac{\hat{\Sigma} : \tilde{D}^p}{(1-f)(1-\phi)(Th + (T - \tau)\Sigma_m)/((1-f)(1-\phi))} \]  
(98)

The void evolution laws can be derived as classically from the energy compatibility condition:

**Table 3**
Values of the parameters of the non-associated model for COx argillite.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Material</th>
<th>Elastic parameters</th>
<th>Plastic parameters</th>
<th>Inter-porosity, ( \phi ) (%)</th>
<th>Intra-porosity, ( f ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Clay</td>
<td>5.027</td>
<td>0.33</td>
<td>23.75</td>
<td>1.0</td>
</tr>
<tr>
<td>1</td>
<td>Calcite</td>
<td>95</td>
<td>0.27</td>
<td>20.60</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>Quartz</td>
<td>101</td>
<td>0.06</td>
<td>20.60</td>
<td>1.0</td>
</tr>
</tbody>
</table>

\[ E \text{(GPa)} \quad \rho \quad T_0 = 0.05, T_m = 0.9, b_1 = 220 \]
4.3.3. Experimental verification

The proposed non-associated micromechanical constitutive model is now applied to reproducing the macroscopic responses of the COx argillite in uniaxial and triaxial compression tests performed on samples with different mineralogical compositions. The elastic and plastic parameters are identified and given in Table 3 by using the same strategy as that proposed in Shen et al. (2012a). The plastic parameters are identified from a uniaxial compression test on a sample from the depth of 466.8 m with 51% of clay matrix, 26% of calcite and 23% of quartz. Then the same group of parameters is

\[
\begin{align*}
\dot{f} &= (1-f) \frac{\dot{\sigma}_m + \dot{\sigma}_s}{\sigma_m + \sigma_s} - (1-f) \frac{\dot{\sigma}_m}{\sigma_m} \\
\dot{\phi} &= (1-\phi) \frac{\dot{\sigma}_m + \dot{\sigma}_s + \dot{\sigma}_b}{\sigma_m + \sigma_s + \sigma_b} - (1-\phi) \frac{\dot{\sigma}_m + \dot{\sigma}_s}{\sigma_m + \sigma_s} 
\end{align*}
\]  

(99)

Fig. 12. Comparison between the numerical simulation and the experimental data of uniaxial compression test. \(f_0 = 51\%, f_1 = 26\%, \) and \(f_2 = 23\%, \) where \(f_0, f_1, \) and \(f_2\) are the contents of clay matrix, calcite and quartz, respectively.

Fig. 13. Comparison between the numerical simulation and the experimental data of triaxial compression test with the confining pressure of 5 MPa. \(f_0 = 44\%, f_1 = 33\%, \) and \(f_2 = 23\%.

Fig. 14. Comparison between the numerical simulation and the experimental data of triaxial compression test with the confining pressure of 10 MPa. \(f_0 = 55\%, f_1 = 23\%, \) and \(f_2 = 22\%.

Fig. 15. Comparison between the numerical simulation and the experimental data of triaxial compression test with the confining pressure of 10 MPa. \(f_0 = 60\%, f_1 = 26\%, \) and \(f_2 = 14\%.

4.3.3. Experimental verification

The proposed non-associated micromechanical constitutive model is now applied to reproducing the macroscopic responses of the COx argillite in uniaxial and triaxial compression tests performed on samples with different mineralogical compositions. The elastic and plastic parameters are identified and given in Table 3 by using the same strategy as that proposed in Shen et al. (2012a). The plastic parameters are identified from a uniaxial compression test on a sample from the depth of 466.8 m with 51% of clay matrix, 26% of calcite and 23% of quartz. Then the same group of parameters is
used for all other uniaxial and triaxial compression tests with different confining pressures (5 MPa and 10 MPa). Fig. 12 shows the comparison between experimental data and numerical results for uniaxial compression test on the sample with 51% of clay matrix, 26% of calcite and 23% of quartz. The cases of numerical results for uniaxial compression test on the sample with 

\[ k \]

Porosity at the mesoscale

Microscopic stress

with the increase of deviatoric stress, effects of both inter-particular and intra-particular pores, as well as the influence of volume fraction of mineral grains. The evolutions of inter-particular and intra-particular porosities in the clay matrix are shown in Figs. 16 and 17, respectively, for samples at the depth of 482.2 m with different confining pressures. It is worth to notice that even if the variation of porosity remains quite small, the plastic behavior of the clay matrix is significantly affected by the presence of inter-particular and intra-particular porosities because of the dependence of the yield function and plastic potential on the both porosities.

5. Conclusions

The present study has concerned the micromechanics-based constitutive models for ductile porous geomaterials taking into account the effects of pore and inclusion on the macroscopic behavior. These micro-macro models take into account the effects of pore and inclusion on the macroscopic constitutive models for ductile porous geomaterials taking into account the effects of pore and inclusion on the macroscopic behavior. These micro-macro models take into account the effects of microstructure and significantly improve the phenomenological ones. For simplicity, porous materials containing one population of pores with different types of matrices (von Mises, Green type, Mises—Schleicher and Drucker—Prager) are studied firstly for the macroscopic criterion. Then, it is extended to a more complex case of double porous media with two populations of pores at different scales. The solid phase at the microscale is assumed to obey to a Drucker—Prager type criterion. Based on these macroscopic criteria, complete constitutive models are formulated and implemented to describe the macroscopic responses of typical porous geomaterials (sandstone, porous chalk and argillite). The accuracies of these micromechanics-based constitutive models are assessed and validated by comparisons between numerical predictions and experimental data with different confining pressures or different mineralogical composites.

Conflict of interest

The authors wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma )</td>
<td>Macroscopic stress field of the porous material</td>
</tr>
<tr>
<td>( \sigma_f )</td>
<td>Microscopic stress field of the matrix</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Porosity at the microscale</td>
</tr>
<tr>
<td>( \phi_m )</td>
<td>Porosity at the mesoscale</td>
</tr>
<tr>
<td>( \phi_\delta )</td>
<td>The volumetric fraction of inclusion</td>
</tr>
<tr>
<td>( D )</td>
<td>Macroscopic strain rate</td>
</tr>
<tr>
<td>( D_i )</td>
<td>Macroscopic dissipation</td>
</tr>
<tr>
<td>( l )</td>
<td>The plastic multiplier</td>
</tr>
<tr>
<td>( l_{pe} )</td>
<td>The equivalent plastic strain in the solid phase</td>
</tr>
<tr>
<td>( E_b )</td>
<td>The bulk and shear moduli, respectively</td>
</tr>
<tr>
<td>( E, \nu )</td>
<td>Young's modulus and Poisson's ratio, respectively</td>
</tr>
</tbody>
</table>

References


Professor Shao received his Ph.D. degree in 1987 from University of Sciences and Technologies of Lille in France. He is currently an excellent-class professor at this university. He was the director of the Laboratory of Mechanics of Lille (2010–2013), a Changjiang chair professor at Wuhan University (2007–2010), an overseas expert for the Chinese Academy of Sciences (CAS). He received an overseas outstanding young investigator award from the NSFC and the CAS. Since 2010, he is a Thousand-Talent chair professor at Hohai University, China. He has received an “Excellent Contributions Award (2011)” from the International Association for Computer Methods and Advances in Geomechanics (IACMAG).

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