Revisiting the Bjerrum’s correction factor: Use of the liquidity index for assessing the effect of soil plasticity on undrained shear strength

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A B S T R A C T

The undrained shear strength (s_u) of fine-grained soils that can be measured in situ and in laboratory is one of the key geotechnical parameters. The unconfined compression test (UCT) is widely used in laboratory to measure this parameter due to its simplicity; however, it is severely affected by sample disturbance. The vane shear test (VST) technique that is less sensitive to sample disturbance involves a correction factor against the soil plasticity, commonly known as the Bjerrum’s correction factor, \( \mu \). This study aims to reevaluate the Bjerrum’s correction factor in consideration of a different approach and a relatively new method of testing. Atterberg limits test, miniature VST, and reverse extrusion test (RET) were conducted on 120 remolded samples. The effect of soil plasticity on undrained shear strength was examined using the liquidity index instead of Bjerrum’s correction factor. In comparison with the result obtained using the Bjerrum’s correction factor, the undrained shear strength was better represented when \( s_u \) values were correlated with the liquidity index. The results were validated by the RET, which was proven to take into account soil plasticity with a reliable degree of accuracy. This study also shows that the RET has strong promise as a new tool for testing undrained shear strength of fine-grained soils.

1. Introduction

The shear strength of fine-grained soils generally can be divided into two parts as drained and undrained shear strengths depending on whether the pore water pressure dissipates or not. In situ shear strength of soils is recorded almost unexceptionally in undrained conditions. There are other cases such as the short-term stability analysis of slopes requiring the undrained shear strength.

The most common tool used to measure the in situ undrained shear strength is the field vane shear test (VSTf). The laboratory techniques for this test briefly include the unconsolidated-undrained test (UUT), unconfined compression test (UCT), direct shear test (DST) and vane shear test (VSTl).

The VST technique was originally developed by the British Army to measure the cohesion of clay sediments, which is the shear strength in certain special cases (Skempton, 1949; Boyce, 1983). The laboratory version of VST, or the miniature VST, may be used to obtain the undrained shear strength of fine-grained soils. The test provides a rapid determination method for the shear strength of undisturbed, remolded and reconstituted soils. It is recommended for use on soils with undrained shear strength less than 100 kPa (ASTM D4648–00, 2000). Essentially, the miniature VST is capable of measuring undrained shear strength of soils from a few kPa to about 100 kPa, which roughly represents the plastic range for most fine-grained soils.

Sample disturbance is one of the most important factors influencing the undrained shear strength of fine-grained soils measured by laboratory techniques. The UCT is one of the most common tools used to determine the undrained shear strength of soils. The undrained shear strength determined in this way is highly sensitive to disturbance caused by the sampling process, compared to other means such as consolidation or triaxial tests (Tanaka et al., 1992; Lacasse et al., 1994; Tanaka, 1994). Based on the conclusion that the undrained shear strength obtained from VSTl is nearly the same as that from VSTf, Tanaka (1994) showed that the vane shear strength is not significantly influenced by the mechanical disturbance caused by sampling, release of overburden pressure, or increase in the confining pressure when the vane is inserted. This important conclusion can be considered as the basis of the VST’s superiority over the UCT, and thus the underestimation of undrained shear strength caused by the sample disturbance during the UCT can be prevented.

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Various researchers have pointed out that the undrained shear strength determined by the VST is influenced by various factors, such as over-consolidation ratio (OCR) (e.g. Jamiołkowski et al., 1985), pre-consolidation pressure (e.g. Skempton, 1957; Larsson, 1980), and particularly the soil plasticity.

Skempton (1957) found that the ratio of undrained shear strength to vertical effective stress of normally consolidated clays is a linear function of the plasticity index for the VST:

\[
\frac{s_u}{\sigma'_v} = 0.11 + 0.0037Pl
\]

where \(s_u\) is the undrained shear strength, \(\sigma'_v\) is the vertical effective stress, and \(P_l\) is the plasticity index. Watson et al. (2000) examined factors such as rotation rate and waiting time that may influence VST results. They also introduced a new form of ‘helical’ VST in which the standard vane apparatus is continuously rotated during penetration into the soil sample, resulting in a complete profile of soil strength.

Bjerrum (1954) stated that the normally consolidated Norwegian clays show a linear increase in undrained shear strength with depth, which can be expressed by a constant ratio of shear strength to effective overburden pressure. If this ratio is determined for various clays, a close correlation can be found between the ratio and the plasticity index.

Bjerrum (1973) proposed a correction factor \(\mu\) (Fig. 1) for shear strength obtained from the VSTF, based on many failure cases. In his proposal, as the effects of anisotropy and strain rate on the shear strength were considered, the factor \(\mu\) can be determined from the plasticity index. This correction factor is still in use to date.

Tanaka (1994) stated that the shear strength modified by Bjerrum’s correction factor is considerably conservative for Japanese marine clay, in comparison with the unconfined compressive strength. In this regard, Dolinar (2010) pointed out that the normalized undrained shear strength can be correlated with the plasticity index \(P_l\) for non-swelling clays, while in swelling clays, the plasticity index does not influence the undrained shear strength of normally consolidated soils. Thus, there is no uniform criterion to determine the normalized undrained shear strength from the plasticity index for all fine-grained soils. Kayabali and Tufencki (2010) stated that, although the VSTL provides a reasonable undrained shear strength value at the plastic limit \((P_l)\), it overestimates the undrained shear strength at the liquid limit \((L_l)\). They recommended that care should be taken when the laboratory VST is used to determine the undrained shear strength at water contents near the liquid limit.

In an attempt to compare natural plastic clays to remolded ones, Graham and Li (1985) examined how the general concepts of soil behavior developed from remolded samples can be applied to samples of a complex, natural clay. They found that, in terms of stress—strain behavior, strengths and yielding, the natural and one-dimensionally consolidated remolded samples of Winnipeg clay show similar though not identical results. In this investigation, we consider this conclusion and the similarity between the undrained shear strengths obtained from VSTL and VSTF on remolded fine-grained soils as noted by Tanaka (1994), the results of which are intended to be applied to fine-grained natural soils.

Kayabali and Ozdemir (2013) proposed the reverse extrusion test (RET) as an alternative to the UCT. By applying this technique on 60 remolded and 75 natural soil samples, they showed that the RET yields a consistent ratio of 14–15 for the extrusion pressure over the unconfined compressive strength. They stated that the RET is likely to eliminate the difficulties involved in the testing of soft to very soft soils, as well as the strength anisotropy for fissured soils using the conventional UCT owing to the confinement provided by the testing apparatus. In conclusion, they pointed out that the RET can better represent the undrained shear strength of soil than the UCT, and that the results may be improved further by taking into account soil plasticity.

The scope of this investigation is to revisit Bjerrum’s correction factor in assessing the effect of soil plasticity on the undrained shear strength of fine-grained soils by using a different approach, as well as employing a relatively recent technique to validate the use of the proposed alternative.

2. Materials and methods

The soil samples used in this investigation include remolded soil samples brought from different regions of Turkey to the university laboratory. All samples are oven-dried, pulverized and sieved through a #40 mesh.

The first tool used to assess the undrained shear strength of remolded fine-grained soils is the laboratory miniature vane shear device which is a Wykehem Farrance model WF2350. Regarding test standards, the guidelines of the ASTM D4648–00 (2000) are followed. The starting water content for the remolded samples to be tested using the VSTL is somewhat smaller than the liquid limit. The soil sample is wetted at this water content and mixed homogeneously prior to shearing. The next test is carried out by adding small amount of dry soil sample to the previous wet mixture and the new sample is remixed presumably at slightly lower water content. The VSTL has four torque springs for different levels of soil stiffness. The appropriate spring is selected for each test so that shear failure will occur between 20° and 90° of sample rotation. The test is repeated 10 times for each soil sample at different water contents. A plot showing the relationship between undrained shear strength \(s_u\) and water content \(w\) obtained from the VSTL is presented in Fig. 2 for sample No. 12.

The second tool used in the investigation is the reverse extrusion device whose principle was first introduced by Whyte (1982). While more details can be found in Kayabali and Tufencki (2010) for this new test in soil mechanics, a brief summary is presented here for convenience. The container has an inner diameter of 38 mm. The top cap is removable for sample extraction after the test is finished. The rammer has a die orifice of 6 mm, which controls the plastic failure of the sample. There is a small clearance between the rammer and the container to prevent metal friction between parts. A general view for this simple setup is shown in Fig. 3. The homogeneous wet mixture of remold soil is placed in the container and tapped gently with the rammer. The height of sample inside
The container is usually kept around 5 cm. The container is placed into a load frame and steady compression is applied. A loading rate of 1 mm/min is usually preferred. The operator watches the display of the compression machine and records the force at certain time interval. As the loading progresses, the sample inside the container is compressed and the compressive force continually increases. When the plastic failure occurs, the soil extrudes from the die orifice and the compressive force is kept constant. A plot showing the relationship between extrusion force \( F_E \) and rammer displacement \( d \) obtained from the RET at different water contents of soils is illustrated in Fig. 4 for sample No. 12. Data pairs of extrusion pressure at failure and the corresponding water content are plotted in Fig. 5. The reader may notice the difference between the horizontal axes of Figs. 2 and 5 for sample No. 12. The water content in the latter one does not extend down to liquid limit. It is attributed to the character of the RET, which becomes very difficult to conduct when the water content is high, particularly when the liquidity index is greater than 0.5.

The liquid limit tests are performed in accordance with the BS 1377 (1990). At least 5 levels of water contents are tried in order to catch the best match between data points and the fitting curve. The plastic limit tests are run following the guidelines of ASTM D427–98 (1998). Five to ten trials are employed per soil sample; afterwards, the mean value is taken by dropping the recorded highest and lowest plastic limit values.

### 3. Experimental results

The results of Atterberg limits tests on 120 samples using the fall-cone and bead-rolling with the rolling plate method are presented in Table 1. As stated earlier, the VSTs are conducted at 10 levels of water contents. The evaluation of the VST results in the form of a graphical plot provides the best relationships when both the axes are linear. It is found that the best empirical form for the dependent variable of undrained shear strength \( s_u \) in kPa in terms of water content \( w \) is as follows:

\[
s_u = ae^{-bw}
\]

where \( a \) and \( b \) are the fitting coefficients.

Presentation of all VST data in terms of water content and undrained shear strength is not practical owing to space limitations. Therefore, the results of VSTs are provided in terms of coefficients \( a \) and \( b \) for each soil sample as listed in Table 1. Excluding a few soil samples, the regression coefficients \( R^2 \) between water content and undrained shear strength are all greater than 0.95 (Table 1).

In order to examine the effect of soil plasticity on undrained shear strength obtained from the VST method, three different approaches are examined: (i) \( s_u \) as a function of water content and the plasticity index \( PI \); (ii) \( s_u \) as a function of water content and Bjerrum’s correction factor, \( \mu \); (iii) \( s_u \) as a function of liquidity index \( LI \) only.
A multiple regression analysis is performed between water content, plasticity index and undrained shear strength for VST data. Data sets comprising those two independent variables and one dependent variable for each sample are included in the regression analysis. The multiple regression analysis is carried out using a VST correlation between only the liquidity index ($\theta_I$) and the Bjerrum's correction factor for soil plasticity, $\mu$, has the empirical form (as deduced from Fig. 1) as

$$\mu = 2.131(1 + PI)^{-0.265}$$

The second step of the multiple regression analysis involves the prediction of undrained shear strength using the independent variables of water content and plasticity index corrected by the Bjerrum’s correction factor. The resulting regression coefficient is 0.71.

The third step of the multiple regression analysis involves the correlation between only the liquidity index ($\theta_I$) and the undrained shear strength. The liquidity index is defined as
LI = \frac{w - PL}{LL - PL} \quad (4)

The multiple regression analysis for this third step yields the following relationship with the $R^2$ value of 0.82, which is the highest value among the results of the three approaches:

\[ s_u = 96 \times 0.187^{LI} \quad (5) \]

The three-step multiple regression analysis reveals that, compared with the result obtained using the Bjerrum’s correction factor, the undrained shear strength obtained from the VST is better represented when $s_u$ values are correlated with the liquidity index. Fig. 6 is a plot of $s_u$ versus $LI$ for 972 data pairs. The number of data pairs is 1200 considering that 10 groups of tests are conducted for each sample. We come to realize that some of the tests are run at water contents below plastic limit and thus are excluded from the analyses, because they result in negative values for $LI$.

There should be a reference test to validate the usability of liquidity index to predict undrained shear strength. Earlier, it is stated that the UCT is significantly influenced by sample disturbance. The present investigation employs only remolded samples, and there is no chance of testing remolded samples using the UCT. Those two facts prohibit us from using the UCT for a comparison with the VSTL. Instead, based upon the proposal by Kayabali and Ozdemir (2013) that the RET is superior to the UCT as it eliminates the fissure effects and enables the testing of soft to very soft soils, the RET technique is employed as a reference to validate the results obtained from the miniature VST.

In contrast to the VSTL data, the RET data result in a better correlation between the extrusion pressure and the water content in a semi-logarithmic diagram as shown in Fig. 5. The correlation between the extrusion pressure at failure and the water content is perfect for sample No. 12, and the smallest value of $R^2$ is 0.98 for all samples. The $y$-intercept and the slope of the fitted curve from Fig. 5 and from the plots of other soil samples are determined and listed in Table 1. Because the slope value (or the coefficient $b$) is small, with several decimals, the inverse of the $b$ value is preferred for convenience. The $y$-intercept (the coefficient $a$) and the inverse slope ($b^{-1}$) values determined for 120 samples are listed in Table 1 along with their regression coefficients.

A correlation between the liquidity index and the extrusion pressure at failure is built using all data from the RET. The following relationship with a regression coefficient of 0.81 is obtained:

\[ P_E = 2127(1 + LI)^{-5.33} \quad (6) \]

As far as the regression coefficients for the VSTL and RET methods are concerned, the degree of accuracy for predicting undrained shear strength using the RET is nearly the same as that of the VST. The relationship between the liquidity index and the extrusion pressure is presented in Fig. 7.

Note that the RET is capable of predicting the undrained shear strength for fine-grained soils, the extrusion pressure at failure ($P_E$) needs to be converted to a convenient form in terms of $s_u$, i.e. the undrained shear strength needs to be determined in terms of $P_E$. It should be noted that the VSTL and the RET are not conducted at the same water content; therefore, a list of common water contents for both test methods needs to be constructed as shown in Table 2, which are assigned arbitrarily. For instance, the liquid limit and plastic limit for sample No. 24 are 42.9 and 26.2, respectively. The arbitrary water contents of 30%, 35% and 40% are assigned for this sample and the corresponding undrained shear strength and extrusion pressure are computed using Eqs. (5) and (6), respectively. Likewise, the $LI$ and $PL$ for sample No. 57 are 90.3 and 35.2, respectively. The arbitrary water contents of 40%, 50%, 60%, 70%, 80% and 90% are assigned and the corresponding undrained shear strengths and extrusion pressures are determined. When the soil’s plasticity index is low, the water content increment is taken as 5% (as in the case of sample No. 24), whereas when the PI is great the increment is selected as 10% (as in the case of sample No. 57).

Table 2 is given only as an example, which also includes coefficients.

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**Table 2**

A sample table to show how the common water contents for the miniature VST and RET, the corresponding liquidity index, extrusion pressure and undrained shear strength are generated.

<table>
<thead>
<tr>
<th>No.</th>
<th>LI</th>
<th>PL</th>
<th>$a_{RET}$</th>
<th>$1/b_{RET}$</th>
<th>$a_{VST}$</th>
<th>$b_{VST}$</th>
<th>w (%)</th>
<th>$LI$</th>
<th>$P_E$ (kPa)</th>
<th>$s_u$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>42.9</td>
<td>26.2</td>
<td>6.1</td>
<td>9.5</td>
<td>790</td>
<td>0.081</td>
<td>30</td>
<td>0.23</td>
<td>896</td>
<td>70</td>
</tr>
<tr>
<td>24</td>
<td>42.9</td>
<td>26.2</td>
<td>6.1</td>
<td>9.5</td>
<td>790</td>
<td>0.081</td>
<td>35</td>
<td>0.53</td>
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<td>46</td>
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<td>26.2</td>
<td>6.1</td>
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<td>0.081</td>
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<td>0.83</td>
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<td>25</td>
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<td>5.1</td>
<td>16.6</td>
<td>393</td>
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<td>30</td>
<td>0.07</td>
<td>1831</td>
<td>88</td>
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<tr>
<td>25</td>
<td>65.2</td>
<td>27.2</td>
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<td>0.34</td>
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<td>90.3</td>
<td>35.2</td>
<td>5</td>
<td>23.1</td>
<td>589</td>
<td>0.045</td>
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<td>1855</td>
<td>97</td>
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<td>57</td>
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<td>0.99</td>
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</table>
\[ s_u = 46.3 \times 0.307^{L_I}P_{E}^{105} \] (7)

4. Conclusions and discussion

The following conclusions are drawn from the present investigation:

1. While the Bjerrum’s correction factor has been well known and in use for a long time, the effect of soil plasticity on undrained shear strength of fine-grained soils is shown to be better represented when the measured undrained strength values are correlated with the liquidity index.

2. The results of the VST1 are validated by the RET, which is shown to be simple, robust, free of operator effects, and to eliminate the two important setbacks with the UCT, indicated by the previous researchers.

3. The RET predicts the undrained shear strength of fine-grained soils with nearly the same degree of accuracy as the VST method. Its use is much simpler than the VST method.

It should be kept in mind that the range of LL of the soil samples in this investigation is 46–91, and the range for PI is 19–57. The equations developed to predict the undrained shear strength using the VST and RET techniques are valid only for those ranges of plasticity. Further investigation is recommended to include soils of both the lower and higher liquid limits than those used for this investigation.

Conflict of interest

The authors confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

References


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