1. Introduction

Permeability is generally defined as the ability of fractured porous media to allow the passage of fluid (Friedman, 1977). It has been significantly influenced by geotechnical and geological engineering activities. Thus, the ability to capture the evolution of permeability under various (mechanical, chemical and thermal) conditions is crucial to several applications in reservoir, geotechnical, mining and petroleum engineering (Berkowitz, 2002).

Several theoretical and experimental investigations have been conducted to characterize permeability evolution laws in terms of porosity, stress, temperature, chemical process, mass removal and failure models (Zhu and Wong, 1997; Morris et al., 2003). Generally, there are four main families of permeability evolution models, i.e. based on (i) porosity, (ii) stress and damage, (iii) equivalent channel concept, and (iv) network model. However, due to the complex interactions between flow and deformation in geotechnical and geological engineering activities, the models mentioned above have their own limitations and can only be applicable for certain conditions. Therefore, studies on permeability evolution models of fractured porous media are highly required to deal with fluid flow problems. Undoubtedly, the understanding of state-of-the-art permeability evolution model would help researchers and engineers develop, modify and apply permeability model through an appropriate approach.

The main objective of this paper is to review the recent advancement of permeability evolution model for fractured porous media. Permeability evolution models proposed by earlier and recent researchers are discussed, with their main features and limitations being noted.

2. Permeability evolution models based on porosity

Changes in permeability and porosity coincide in laboratory experiments, and quite a few theories have been proposed to investigate the relationship between these two parameters (Zhang et al., 1994b; Zhu and Wong, 1997; Schutjens et al., 2004; Zhu et al., 2007; Hu et al., 2010). Based on experimental observations, Sulem and Ouffroukh (2006) indicated that the permeability of porous media is strongly influenced by the initial porosity, stress level, deformation process (e.g. strain hardening-compaction and strain softening-dilatancy), pore geometry and structure. Generally, two main approaches can be identified among the existing models for permeability-porosity relationship: the exponential function model and the power function model. Among them, the most widely accepted approach is the generalized power law, which is formulated in the permeability-porosity space, log-log space and semi-log space (David et al., 1994; Bernabé et al., 2003; Morris et al., 2003; Zhu et al., 2007). The other approach, i.e. the exponential law, was proposed by David et al. (1994), and it was initially applied to simulating the compaction-induced permeability reduction. This approach was also adopted by Zhu and Walsh (2006) and Zhu et al. (2007) to demonstrate the relationship between permeability and mean stress prior to the onset of shear-enhanced compaction or...
dilatancy. The main features, as well as the limitations of such two approaches, are discussed in the following subsections.

2.1. Permeability-porosity models based on power law

The Kozeny-Carman (KC) model (Kozeny, 1927; Carman, 1937) is one of the most widely accepted and simple models that links permeability and porosity in a general loading space. It can be expressed as (Walsh and Brace, 1984)

\[ K = \frac{\phi^3}{B \sqrt{S}} \quad (1) \]

where \( K \) is the permeability of porous media; \( \phi \) is the porosity of porous media; \( \tau \) is the tortuosity defined as the ratio of real flow path to the straight path from flow-in-point to flow-out-point; \( S \) is the specific surface area (surface area per unit volume) of porous media; \( B \) is the pore shape coefficient, which is 2 for circular tubes and 3 for thin cracks. This model provides reasonable simulation results under certain conditions; however, it cannot conveniently be used because of the specific surface area parameter \( S \) and the pore shape coefficient \( B \) which are not easily calibrated. Several semi-empirical equations have subsequently been proposed to improve the estimation of rock permeability subjected to various loading conditions (Panda and Lake, 1994; Bernabé et al., 2003; Costa, 2006; Zhu et al., 2008); and some modified KC models are listed here.

Bayles et al. (1989) proposed a porosity-permeability relationship based on the fractal pore cross-sectional area, which can be formulated as

\[ K = c \left( \frac{\phi^z}{1-\phi} \right)^2 \quad (2) \]

where \( c \) is a constant to determine permeability, and \( z \) is an exponent parameter for porosity. A similar permeability formulation based on fractal pore space observations developed by Costa (2006) is written as

\[ K = c \left( \frac{\phi^z}{1-\phi} \right)^2 \quad (3) \]

Another empirical KC-like permeability-porosity model for glass and fiber mats conducted by Rodriguez et al. (2004) is expressed as

\[ K = c \left( \frac{\phi^z}{1-\phi} \right)^2 \quad (4) \]

Additional simple and empirical functions can be identified within this family. For instance, using a single transient test, Ghabezloo et al. (2009) adopted a power law to analyze the low-permeability creeping material:

\[ \frac{K}{K_o} = \left( \frac{\phi}{\phi_o} \right)^\alpha \quad (5) \]

where \( \alpha \) is the porosity sensitive exponent that depends on the properties of the material and on the evolution process; and \( K_o \) and \( \phi_o \) are the initial permeability and porosity, respectively. Based on experimental observations, David et al. (1994) suggested that the porosity sensitive exponent \( \alpha \) ranges from 1 to 25 for common geological materials. For instance, the exponent \( \alpha = 11 \) in the analysis by Ghabezloo et al. (2009). Zhang et al. (1994a) conducted in-situ tests to measure the permeability during hot pressing of calcite, and the measurements showed that, in the relatively high porosity regime, the permeability changes with porosity following a power law with an exponent of 3, which is in agreement with the work of Zhu et al. (1995) and Lockner and Evans (1995). Experimental data for Berea and Boise sandstone showed that the exponent \( \alpha \) increases with increasing effective mean stress; \( \alpha = 19.5 \) for Boise sandstone in the cataclastic regime, whereas for the Darley Dale stone, a low porosity rock with a porosity of 14%, \( \alpha \) decreases from 19.5 to 11.3 (Zhu and Wong, 1997; Bernabé et al., 2003). Therefore, \( \alpha \) depends on both the confining stress level and the initial porosity (Bernabé et al., 2003).

Another well-established approach is based on the concept of percolation, the hypothesis of which is expressed as: the pore connectivity will vanish if the porosity is below a certain level, known as the percolation threshold, \( \phi_{cr} \). Therefore, some investigators have suggested that it would be more appropriate to address the permeability-porosity relationship by considering percolation theory (Diens, 1983; Zhang et al., 1994b; Guéguen et al., 1997; Alkan, 2009). Applying this concept, Sahimi (1994) proposed a power law of permeability-porosity:

\[ K = c(\phi - \phi_{cr})^2 \quad (6) \]

This model is simple and has been the subject of several investigations. For instance, Sornette (1987) and Feng et al. (1987) suggested \( z = 4.4 \) and \( \phi_{cr} \) with values of 0.0026—0.036 using the “Swiss-Cheese” model; Zhang et al. (1994b) used \( \phi_{cr} = 0.04 \) and \( z = 2.18 \) to fit the experimental data of various materials. Saar and Manga (1999) indicated that, for the fully penetrable sphere (FPS) model, \( \phi_{cr} = 0.3 \) and \( z = 2 \), which were also proposed by Feng et al. (1987) and Sahimi (1994, 1996). However, the main limitation of this model is that it requires several experimental tests to calibrate the model parameters, which are based on measuring fractal properties of the solid matrix system. Also, these parameters were obtained through curve-fitting based on experimental observations of specific materials, thus limitation may be arisen to other engineering materials, in which pore spaces, tortuosity, interfacial spaces and other internal geometries are generally quiet different. However, due to their simplistic form and limited number of parameters, they are still preferable in engineering application.

2.2. Permeability-porosity models based on exponential law

Another form of the permeability-porosity relationship is expressed by exponential functions. In studying the basin drill core, Nelson (1994) and Bethke (1985) proposed an empirical equation, which suggests that the porosity varies linearly with the log of permeability:

\[ \log_{10}K = c_1\phi + c_0 \quad (7) \]

where the coefficients \( c_1 \) and \( c_0 \) are derived from the regression fitting of laboratory data for the core. Other investigators determined values of \( c_1 \) and \( c_0 \) for sand (Bethke, 1985), shale (Neuzil, 1994; Garavito et al., 2006), and carbonate rock reservoirs (Lucia and Fogg, 1990). Morris et al. (2003) fitted the experimental data of rocks (Zhu and Wong, 1997) with a porosity greater than 16.7%; the permeability-porosity relationship takes the following form:

\[ K = K_0 \exp(C\phi) \quad (8) \]

where \( C \) is the permeability-porosity exponent. It is observed that upon further reduction in porosity (less than 16.7%), a sudden reduction in the permeability occurs due to irreversible damage (Zhu and Wong, 1997). Morris et al. (2003) approximated this process by
\[ K = K_0 \exp[\psi(p', d_p) - d_p \min(d_p, d_{p\text{max}})] \] (9)

where \( p' \) is the effective mean stress; and \( d_p \) and \( d_{p\text{max}} \) are the permeability damage reduction factor and the maximum permeability damage, respectively. This equation is based on the observation that a section of the permeability-porosity curve becomes a straight line, which signifies the reduction in permeability due to damage (Morris et al., 2003).

Yang and Aplin (2010) proposed a linear relationship between the log of permeability and the porosity for mudstone over a restricted regime:

\[ \ln K = a_K + b_K e + c_K e^{0.5} \] (10)

where \( e \) is the void ratio; \( a_K, b_K \) and \( c_K \) are the coefficients (in \( m^2 \)) that are functions of the clay content. As indicated by Yang and Aplin (2010), a slightly more complex formulation could better describe the permeability-porosity relationship over the full range of porosity in mudstone. Similar treatment of the logarithmic permeability and porosity has also been found in other studies (Nagaraj et al., 1994; Dewhurst et al., 1999; Yang and Aplin, 2007).

In general, \( KC \) family of models describes the evolution of permeability through a fitting process, which lacks a mechanical or geometrical analysis of the problem. Therefore, they cannot be used universally without modification. Nevertheless, empirical approaches are still preferred for engineering applications due to simplicity of the approach and limited number of parameters involved. In addition, the \( KC \) family of models is flexible and can be easily modified or specified by incorporating additional mechanical or geometrical parameters related to the main features of the material studied.

3. Permeability functions based on the stress and damage concept

Rice (1992) proposed an exponential permeability-stress relation when analyzing the generation and dissipation of excess pore pressure in seismogenic systems. The exponential law takes the following form:

\[ K = K_0 \exp\left(-\frac{p}{p_0}\right) \] (11)

where \( p \) is the normal stress, and \( p_0 \) is the reference normal stress taken as 5 MPa.

David et al. (1994) also introduced a similar relationship except that a pressure sensitivity coefficient \( r \) is adopted and \( p_0 \) is taken as equal to the reciprocal of \( r \) under hydrostatic loading. However, the physical meaning and measurement of \( p_0 \) are still unclear.

Lyakhovsky and Hamiel (2007) proposed a power law that accounts for the damage variable:

\[ K = K_0 \exp\left(\frac{\psi}{\varphi_0} \cdot z \cdot \left(\frac{D}{D_0}\right)^z\right) \] (12)

where \( D \) and \( D_0 \) are the current damage variable and the reference damage value, respectively; and \( z' \) is the exponent parameter for damage. By fitting the experimental data from Tenhovey et al. (1998), \( z = 2 \) is obtained from the numerical solution, and \( z' = 3 \) is obtained for the permeability-porosity relationship.

David et al. (1994) plotted the effective permeability pressure in a semi-log plot as a function of the compaction pressure below the shear-enhanced compaction or dilatancy. They approximated the compaction-induced reduction in permeability as an exponential function:

\[ K = K_0 \exp[-r(p' - p_0)] \] (13)

Based on experimental work, Zhu (2006) demonstrated that, prior to reaching a critical effective mean stress \( C' \), the exponential law provides a good estimation of the relation between permeability and porosity. He proposed a probabilistic damage model to characterize the evolution of permeability during shear-enhanced compaction. Zhu et al. (2007) extended this model to quantify the stress-induced anisotropy in the permeability during cataclastic flow:

\[ \ln K = \ln K_0 - r(p' - p_0) - \frac{\beta}{2} \left[ 1 + \text{sign}(p - j) \cdot \text{erf}\left(\frac{b - j}{\delta}\right) \right] \] (14)

where \( \beta \) is a multiplier coefficient; \( j \) and \( \delta \) are the slight and mean percentages of the Gaussian distribution, respectively. The \( \text{sign}(p - j) \) is positive if \( p > j \) and is negative if \( p < j \) before the error function. The error function \( \text{erf}\left(\frac{b - j}{\delta}\right) \) is introduced to very small if the effective mean stress is below the critical stress \( C' \), and the effect of the deviatoric stress becomes negligible; if the effective mean stress is in the vicinity of \( C' \), the error function changes rapidly, which means that the shear stress may dominate in this deformation regime.

Similarly, Tang et al. (2002) proposed a coupled flow-stress-damage (FSD) model for rocks by extending Biot's theory to include the effects of stress and damage on permeability:

\[ K = \begin{cases} K_0 \exp[-\gamma(p - \chi p_0)] & (D = 0) \\ \omega K_0 \exp[-\gamma(p - \chi p_0)] & (0 < D \leq 1) \end{cases} \] (15)

where \( \omega \) is the damage factor of permeability which is greater than 1, and it is defined as the increase in permeability caused by damage; \( \chi \) and \( \gamma \) are the coefficients defined in Biot's seepage equation and permeability-stress relation, respectively. The results of this model show that the nature of fluid flow in rocks varies from material to material and that heterogeneity significantly affects flow features. The FSD model proposed by Tang et al. (2002) is formulated in two-dimensional (2D) space. As an extension, Li et al. (2010) conducted a three-dimensional (3D) FSD model to study the permeability-stress evolution for the pre-failure and post-peak stress stages of rock at an elemental scale; and the failure process and fluid flow are investigated in a large-scale element. Note that the damage effects taken into account from Eqs. (15) and (12) are totally different: \( \omega \) (Eq. (15)) accounts for the damage-induced increment in permeability, and it does not mean that permeability increases with increasing damage, which makes the FSD model different from Eq. (12) by Lyakhovsky and Hamiel (2007).

The permeability evolution laws based on stress and damage concepts combine semi-empirical and semi-mechanical approaches. This approach provides a good estimation of the variation in permeability for materials subjected to mechanical changes or to damage, which does not take into account the deformation variables. These models also enjoy a greater level of flexibility. They are complex when considering mechanical or geometrical parameters, or can be made simple under certain conditions when adopting an empirical approach to a specific condition. Despite these advantages, the lack of geometrical and mechanical parameters has limited their application to practical problems.

Recently, due to rapid advance in industry of coal seam gas and unconventional energy exploitation, stress-induced permeability evolution models in fractured sorbing media and multiphase flow media are investigated extensively. Notable work published recently includes: Robertson and Christiansen (2007), Zhang et al. (2008), Clarkson et al. (2008), Liu and Rutqvist (2010), Liu et al. (2011a), Wang et al. (2012), Bedayat and Taleghani (2012), Arson and Pereira (2013), Mokhtari et al. (2013), Cho et al. (2013),
Latham et al. (2013), Peng et al. (2014), Wang et al. (2014), etc., with major characteristics and development of this group of models being discussed and investigated by Liu et al. (2011b). Due to extensive discussion and comparison made by many investigators (Liu et al., 2011b; Wang et al., 2012; Latham et al., 2013; Mokhtari et al., 2013), details of this kind of models are not presented, with brief summary being given herein. The advancement of these models is the high accuracy to describe the variation of permeability due to pressure/stress around fractures explicitly and to account for the deformation process. This is attributed to careful considerations of poromechanical responses and sorption-induced deformation inside pore matrix, which also are addressed by various approaches through different viewpoints. Application of these models may encounter some difficulties: parameter identification, real in-situ stress conditions of coal seams/shale reservoirs and coupling with deformation in numerical analysis.

4. Equivalent channel models

Paterson (1983) proposed an equivalent channel model for permeability evolution (see Fig. 1), which can be expressed as

$$K = \frac{BR^2}{F}$$

(16)

$$F = \frac{(l/c)^2}{\phi}$$

(17)

where $R$ is the hydraulic radius; the shape coefficient $B$ of a pore is 1/2 for a circular cross-section, 2/3 for an equilateral triangular cross-section, and 1/3 for a slot; $(l/c)^2$ is a relative tortuosity factor, in which $l$ is the real flow path and $c$ is the straight path from flow-in-point to flow-out-point; $F$ characterizes the entire porous body as a channel; and $BR^2$ reflects the cross-section of the equivalent channel as well as its resistance to fluid flow (see Fig. 1).

Walsh and Brace (1984) proposed an equivalent channel model, which can be expressed as

$$K = \frac{(\phi/S)^2}{FB}$$

(18)

where

$$F = \tau^2 / \phi$$

(19)

These two equivalent channel models are quite similar; therefore, most authors credit the equivalent channel model to both Paterson (1983) and Walsh and Brace (1984). This model can provide useful insight into the relation between the pore geometry and transport properties. However, it assumes that there are no preferential paths and that the hydraulic flow paths are identical to the electrical flow paths, which are not realistic assumptions (Fredrich et al., 1993; David et al., 1994; Zhu et al., 1995).

5. Pore network models

A pore network, as shown in Fig. 2 (conceptual model) and Fig. 3 (simplified model), consists of a series of nodes that represent the individual pores of the pore structure and the bones that link the nodes of the neighboring pore spaces (Van Marcke et al., 2010).

The conceptual model developed by Bernabé (1991) includes three types of pore tubes: nodal pores, tabular pores, and sheet-like conduits. Zhu et al. (1995) proposed a simplified network model by choosing a pipe with a circular cross-section as the conducting element.

The permeability of porous media is determined by the pore space geometry, such as the pore size distribution and the connectivity of the pores (Bernabé, 1991). Fatt (1956) introduced a network model of fluid flow in porous media and used a 2D lattice of pore spaces to investigate the permeability evolution in drainage; however, a 2D regular model may cause a very irregular and complex geometry for the description of a 3D pore space (Van Marcke et al., 2010). Thus, several studies have been conducted to estimate the flow properties using regular networks (Chatzis and Dullien, 1977; Blunt and King, 1991; Dixit et al., 1998), but regular networks fail to capture the statistical distribution of the pore space. Therefore, more realistic random networks have been used to characterize the real complexity of the pore structure (Bryant et al., 1993). Zhu et al. (1995) developed a network model with regular topology (cubic), but with a local conductance distributed randomly according to a probability function based on micro-structure measurements. Similar investigations can also be found in the literature (Zhu and Walsh, 2006; Zhu et al., 2008; Algive et al., 2009; Raoof and Hassanizadeh, 2010), Garboczi and Bentz (1996, 1997) used electron microscopy to obtain a 2D micrograph of the pore structure and then measured the particle size distribution to construct a 3D model to simulate the hydration of concrete. More recently, the X-ray computed tomography (CT) technique has been used for applications of flow-deformation in geo-materials; several network models are based on 2D or 3D imagery (Vogel, 2000; Vandersteen et al., 2003; Balhoff et al., 2007; Thompson et al., 2008; Lemarchand et al., 2009; Joekar-Niasar et al., 2010; Raoof and Hassanizadeh, 2010; Van Marcke et al., 2010; Sun et al., 2012; Jivkov et al., 2013; Raoof et al., 2013; van der Land et al., 2013; Ma et al., 2014; Yin and Zhao, 2014), which provide visual images and better understanding of fractured porous media under
with volumetric change and fracture deformation is complicated under various loading conditions. Further modification of these models and development of new permeability evolution models are highly required to deal with flow problems.

Conflict of interest

The author wishes to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

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References


Jianjun Ma received his Doctor degree in Civil Engineering from The University of New South Wales, Australia. He is a Lecture of Geotechnical Engineering at Wenzhou University, China. Before joining Wenzhou University, he was a research associate in The University of New South Wales, Australia, involving in research projects, i.e. ‘CO2 Sequestration in Geoformations’ and ‘Coupled Flow Deformation Analysis of Fractured Porous Media subject to Elasto-Plastic Damage’ and so on. He is doing research and consulting in Geotechnical Engineering, including constitutive model (plasticity theory and continuum damage mechanics) and numerical analysis (coupled flow deformation analysis, FEM and distinct lattice spring model) in Geomechanics.