Development of a time-dependent energy model to calculate the mining-induced stress over gates and pillars

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Abstract

Generally, longwall mining-induced stress results from the stress relaxation due to destressed zone that occurs above the mined panel. Knowledge of induced stress is very important for accurate design of adjacent gateroads and intervening pillars which helps to raise the safety and productivity of longwall mining operations. This study presents a novel time-dependent analytical model for determination of the longwall mining-induced stress and investigates the coefficient of stress concentration over adjacent gates and pillars. The model is developed based on the strain energy balance in longwall mining incorporated to a rheological constitutive model of caved materials with time-varying parameters. The study site is the Tabas coal mine of Iran. In the proposed model, height of destressed zone above the mined panel, total longwall mining-induced stress, abutment angle, induced vertical stress, and coefficient of stress concentration over neighboring gates and intervening pillars are calculated. To evaluate the effect of proposed model parameters on the coefficient of stress concentration due to longwall mining, sensitivity analysis is performed based on the field data and experimental constants. Also, the results of the proposed model are compared with those of existing models. The comparative results confirm a good agreement between the proposed model and the in situ measurements. According to the obtained results, it is concluded that the proposed model can be successfully used to calculate the longwall mining-induced stress. Therefore, the optimum design of gate supports and pillar dimensions would be attainable which helps to increase the mining efficiency.

1. Introduction

With the development of the mechanized longwall mining methods, underground coal mining has been improved from both production and productivity points of view. However, there are still certain risks in mining that can result in unacceptable level of safety. Generally, one of the hot issues relates to roof fall and ground control (Peng, 1986). Knowledge about the level of stress in a rock mass is very important for underground mining, because many underground mining problems involve stress determination in order to provide the safety considerations. However, determination of stress level in the surrounding rock mass of underground openings is a complex task associated with the geological conditions, the mechanical properties of rocks, the state of in situ stresses, and so on. Indeed, design of a system in rock mass needs to answer a number of questions related to the mechanical behaviors of material, such as fracture, yield, fatigue, stress, and creep (Singh and Verma, 2005).

In the case of longwall mining, accurate estimation of stress level in surrounding rock mass is important for the stability analysis and evaluation of optimum shape and support requirements for longwall panels and side entries (Peng, 2006). However, there are a number of factors that influence the stress distribution around a mined panel. Before the longwall panel extraction, the weight of overburden is uniformly distributed over the coal seam with the relatively strong roof and floor rocks. After the coal seams extraction within a considerable panel width, the roof rock strata fractured and then caved. This process causes disturbance of the original equilibrium conditions. However, potential energy balance requires that the stress distribution in the area has to be readjusted in order to reach a new equilibrium state. As a result, a destressed zone occurs in the panel roof rock strata and the load previously supported by the extracted material is transferred to the surrounding gates and pillars (Peng, 1986). Generally, the caved and fractured zones are at least partially destressed. Accordingly, the overburden weight above the combination height of the caved and fractured zones (height of destressed zone) is shifted to the neighboring solid sections. Indeed, the induced stress is determined by the difference between the weight of the overburden and the weight of destressed zone.
There are several methods for evaluation of the stress distribution around a longwall panel, i.e., in situ measurement, laboratory physical simulation, numerical modeling, and analytical modeling. In situ measurements and laboratory physical simulation are time-consuming, difficult, and expensive (Jiráňková et al., 2012). Numerical modeling is a commonly used method to evaluate stress distribution around underground openings. However, it is time-consuming and requires a large number of input variables. The later limitation depends on the available measured data and may need some estimations or assumptions (Suchowerska et al., 2012). Analytical modeling is a simple and inexpensive method that includes the useful and reliable results in most cases.

In the present study, a new time-dependent analytical model is developed to determine the mining-induced stress and the coefficient of stress concentration (ratio of secondary to primary stress) over gates and pillars based on the strain energy balance in longwall mining. For this purpose, the height of destressed zone above over gates and pillars is determined. The theoretical analysis in this research is supplemented by a specific example of model application to calculate the coefficient of stress concentration over gates and pillars in Tabas coal mine of Iran.

2. Stress distribution around longwall panel

Due to the extraction of the coal seam in longwall mining, state of stress equilibrium in the surrounding rock mass is disturbed and the stress redistribution occurs. The portion of the overburden weight over the destressed zone that is not supported by the extracted area (goaf) is transferred to the adjacent solid abutments, such as gates and pillars. This stress is transferred due to longwall mining and called “mining-induced stress”. In the past, a number of attempts to utilize field observations, theoretical, empirical and numerical approaches were made to understand the nature and amount of mining-induced stress variation in and around an underground excavation due to longwall mining. Considering an infinite, elastic, isotropic and homogenous nature of coal measure formations, Salamon (1964) developed an analytical equation to evaluate the stress distribution at the edge of a longwall panel. Whittaker and Singh (1979) concluded that the peak abutment stress ranges from 4 to 5 times the cover load for UK longwall mining conditions. Wilson (1982) studied the extent of the induced stress zone and found that the cover pressure distance for the safer pillar design conditions is equal to 0.3 times the working depth. Peng (1986) showed that the peak abutment stress is in the range of 1.5–5 times the cover load. Based on the field measurement and numerical modeling, Majdi (1988) developed an empirical equation to calculate the coefficient of stress concentration due to longwall mining. Trueman (1990) proved that the distance at which cover pressure is reached in the goaf is the function of extracted seam thickness and bulking factor of the immediate roof rock strata.

Majumder and Chakrabarty (1991) found that the mining-induced stress increases with the decreasing extracted seam thickness. Mark (1990) developed an equation to estimate the abutment load during final extraction of coal based on the concept of abutment angle which is known as analysis of longwall pillar stability (ALPS). Sheorey (1993) showed that the peak stress around the longwall face will reduce to the cover pressure at a distance of 0.12 times the working depth. Taking into account the typical values of geometric parameters and rock mass properties, Heasley (1998, 2000) considered homogenous stratification of overburden to derive an analytical equation in order to calculate the induced abutment stress. Based on the numerical modeling, Törnroth et al. (2002) proved that the coefficient of stress concentration over the roadways is approximately 2.3 times the cover pressure. Yavuz (2004) demonstrated that an increase in mining height causes increase in cover pressure distance. According to field measurements and laboratory investigations conducted by Jayanthu et al. (2004), the maximum vertical stress over rib and stook decreases with increase in working height during depillaring. Based on the numerical modeling, Yasitli and Unver (2005) proved that the front abutment pressure increases to 14.4 MPa and then decreases gradually toward the initial field stress value (5.75 MPa). According to in situ measurements conducted by Ouyang et al. (2009), the maximum mining-induced stress is equal to 17.5 MPa (the related coefficient of stress concentration is equal to 1.85). By using the numerical modeling, Singh and Singh (2010) showed that the maximum induced stress is equal to 12.1–19.9 MPa in different situations.

Yang et al. (2011) studied the stress evolution with time and space during longwall coal mining. They showed that the maximum abutment stress is approximately 1.67–2 times the original one when the coal seam is mined 20–30 m. The finite element modeling performed by Khanal et al. (2011) demonstrated that the maximum induced vertical pressures is approximately 4 times the in situ pre-mining stress. By using the numerical modeling, Yu et al. (2011) found that the vertical stress over the roadway varies from 13 MPa to 17 MPa (the related coefficient of stress concentration is equal to 1.95–2.55). Xie et al. (2011) performed field monitoring using stress sensors and showed that the maximum stress concentration factors of the stope side and tunnel side are 1.24 and 1.08, respectively. Likar et al. (2012) found that the maximum coefficient of stress concentration around a longwall panel is equal to about 1.8, 2.1, and 1.7 obtained from the in situ measurement, mathematical and numerical modeling, respectively. Khanal et al. (2012) proved that the front abutment stress can reach 15 MPa and the vertical stress on chain pillar can be seen to reach as high as 27 MPa. By using the numerical modeling, Song et al. (2012) found that the maximum stress of ore pillar comes up to 13.4 MPa that is 8.5 MPa greater than the original one. Accordingly, the coefficient of stress concentration over the ore pillar is 2.73. Numerical study conducted by Dattatreyulu et al. (2012) revealed that the vertical stress in the chain pillar can be more than 40 MPa, i.e., more than 4 times the in situ pre-mining stress. According to numerical study conducted by Shabanimashcool and Li (2013), the maximum vertical stress in the pillar is about 3.2 times the in situ vertical stress. Based on the numerical study, Jia et al. (2013) indicated that the maximum mining-induced stress over the pillars is 2.87 times the original stress. According to field measurements performed by Guo et al. (2013), the maximum induced principal stress is close to horizontal one and its value is 1.5–1.7 times the gravitational stress. Verma et al. (2013) studied the effect of excavation stages on stress and pore pressure changes for an underground nuclear repository using three-dimensional (3D) finite difference method. They found that the vertical stress changes during different excavation stages. Gao et al. (2014) proved that the maximum induced vertical stress is approximately 61.6 MPa, corresponding to 2.3 times the overburden stress.

According to the above-mentioned works, there are some analytical equations to evaluate the stress distribution around a longwall panel, as shown in Table 1. Also, the coefficients of stress concentration around a longwall panel obtained by the available analytical, numerical and empirical models as well as in situ measurements presented by different researchers are given in Table 2.

3. Energy considerations in longwall mining

Extraction of coal seam in the longwall mining causes disturbance of the energy balance within a system enclosing mine
openings and surrounding rocks. Therefore, the energy transfer takes place in the rock mass during a transition from one equilibrium state to another, which is assumed to be an elastic continuum. In this research, state I is considered as the primary status before the coal seam extraction and state II is considered as the final extraction of coal seam inside a panel (Fig. 1). It is obvious that both in states I and II, the system of forces are in equilibrium. In Fig. 1, H is the depth of cover, H_d is the height of destressed zone, D is the distance of horizontal line across the center of gravity of the panel to the ground surface, V_m is the volume of materials that should be extracted for transition from state I to state II (the volume of mined panel), S_m is the surface of mined panel, V is the volume of rock masses above a longwall panel which are affected and destressed due to mining (the volume of destressed zone above a panel), and S is the surface of destressed zone above a panel.

In the system of longwall mining enclosing the longwall panel and its surrounding rock mass, the equilibrium between its energy components should be kept during mining. Due to extraction of the coal seam within a considerable panel width and after advancing the hydraulic jacks; the immediate roof of the mined panel is unsupported and hence is allowed to collapse and cave in some distance behind the hydraulic jacks or in the goaf area. This leads to release of the stored strain energy in the mined rock (coal), transferring to the panel roof rock strata. The released energy increases the strain of the roof rock strata and causes fracturing, caving and destressing of those layers. This process is continued until the final extraction of a longwall panel. At the end of panel extraction and after thorough compression of caved materials over the time, a defined zone with the height of H_d is formed above a longwall panel namely “destressed zone” (see Fig. 1b). Beyond the height of destressed zone, overburden weight is transferred towards the front abutment, the adjacent gateroads, the intervening barrier pillars as well as the panel ribs.

![Fig. 1](image-url)  
**Fig. 1.** Longwall mining configuration. (a) State I: before the panel extraction; (b) State II: after the final extraction of longwall panel.
Considering the above descriptions, it can be concluded that the total strain energy stored in the mined rock (coal) is released and consumed in fracturing, caving and destressing the panel roof strata. Therefore, the stored strain energy in the mined rock (coal) should be equal to the stored strain energy in the destressed zone:

\[ U_m = U_d \]  

where \( U_m \) is the stored strain energy in the mined rock (coal), and \( U_d \) is the stored strain energy in the destressed zone.

4. Time-dependent energy model

For construction of the time-dependent model to evaluate the mining-induced stress over gates and pillars in this research, the height of destressed zone above the mined panel is firstly estimated in the long-term condition based on the strain energy balance in the longwall mining. After that, total induced stress is determined by the difference between the overburden weight and the weight of destressed zone. Then, vertical component of induced stress is calculated. Height of destressed zone can be obtained by putting 0 to following equation:

\[ \frac{d}{2} = \frac{1}{2} \left( \frac{K_l \sigma_{r0}}{\gamma} \right) \]  

where the term of \( \frac{1}{2} \) is the moment of inertia of surface \( S_m \) with respect to the plane of ground surface (see Fig. 1b).

According to the parallel-axis theorem in statics, we can write

\[ I = I_0 + A_m D^2 \]  

where \( I_0 \) is the moment of inertia of the mined panel cross-section with respect to a horizontal line across its center of gravity (point \( O \) in Fig. 1b), and \( A_m \) is the cross-section area of the mined panel. With regard to the rectangular cross-section of the mined panel (see Fig. 1b), \( A_m \) and \( I_0 \) are calculated as follows:

\[ A_m = L_w h_s \]  
\[ I_0 = \frac{L_w h_s^3}{12} \]

The substitution of Eqs. (8) and (9) into Eq. (7) produces

\[ I = \frac{A_m h_s^2}{12} + A_m D^2 \]  

According to Fig. 1b, we can get

\[ D = H + h_s/2 \]  

Finally, the substitution of Eqs. (10) and (11) into Eq. (6) is simple to obtain the final equation of the total stored energy in the extracted coal seam as follows:

\[ U_m = \left( 1 + \nu \right) \left( 1 - 2\nu \right)^2 H^3 \]  

4.1. Height of destressed zone

To calculate the height of destressed zone \( (H_d) \), stored strain energy in the extracted coal seam \( (U_m) \) and stored strain energy in the destressed zone \( (U_d) \) in the long-term condition should be calculated. Height of destressed zone can be obtained by putting equal of these two mentioned energy components.

4.1.1. Calculation of \( U_m \)

Salamon (1984) derived the following equation to calculate the stored strain energy in the extracted coal layer:

\[ U_m = \frac{1}{2} \int_{S_m} T^p \cdot u^p \, dS - \int_{V_m} X_i u^p_i \, dV \]  

where \( T^p_i \) is the stress or traction vector acting on a surface in state \( I \), \( u^p_i \) is the component of the displacement vector in state \( I \), and \( X_i \) is the body force per unit volume in the rock. The superscript “\( p \)” stands for “primitive” or state \( I \).

Since no permanent supports are used in longwall mining, the effect of body forces can be ignored \( (X_i = 0) \). Therefore, Eq. (2) is modified to following equation:

\[ U_m = \frac{1}{2} \int_{S_m} T^p_i u^p_i \, dS \]  

The traction vector acting on a surface in state \( I \) \( (T^p) \) and only non-zero displacement component \( (u^p_i) \) are calculated as follows (Salamon, 1984):

\[ T^p_i = \sigma_{r0} = \gamma H \]  
\[ u^p_i = \frac{(1 + \nu)(1 - 2\nu)\gamma H^2}{2(1 - \nu)E} \]

The substitution of Eqs. (4) and (5) into Eq. (3) results in

\[ U_m = \int_{S_m} \frac{1 + \nu}{2(1 - \nu)E} \left( 1 - 2\nu \right)^2 H^3 \, dS \]

\[ = \frac{(1 + \nu)(1 - 2\nu)\gamma^3 H^3}{2(1 - \nu)E} \int_{S_m} H^2 \, dA \]  

where the term of \( \int H^2 \, dA = I \) is the moment of inertia of surface \( S_m \) with respect to the plane of ground surface (see Fig. 1b).

4.1.2. Calculation of \( U_d \)

In general, the stored strain energy of caved materials within the destressed zone \( (U_d) \) is composed of elastic strain energy \( (U_E) \) and viscoplastic strain energy \( (U_V) \):

\[ U_d = U_E + U_V \]  

In this research, the rheological properties of the caved materials are considered according to the nonlinear rheological constitutive model introduced by Zhang et al. (2011). To investigate the rheological properties of the caved materials in this model, the elastic modulus \( (E) \), the coefficient of viscosity \( (\mu) \) and the threshold value of stress \( (\sigma_s) \) are considered based on the modified Bingham model (Fig. 2).

The nonlinear rheological constitutive model is described as follows:

\[ \sigma = \begin{cases} E_0 e^{-at} & (\sigma \leq \sigma_s) \\ \frac{\sigma - \lambda \dot{\varepsilon}}{K} & (\sigma > \sigma_s) \end{cases} \]

where \( \sigma \) is the stress of caved materials, \( E_0 \) is the initial elastic modulus, \( a \) is the material constant, \( t \) is the pressure time of caved materials, \( \varepsilon \) is the strain of caved materials, \( \lambda \) is the slope of materials hardening stage, and \( K \) is the coefficient calculated by the following equation:
The initial elastic modulus \( (E_0) \) in Eq. (17) is taken as equivalent of the rock mass elastic modulus \( (E) \). Thus, Eq. (17) is modified to
\[
U_E = \frac{1}{2} E e^{-at} A_d H_d t^2
\]  
(18)

The following equation is also used to calculate the strain of caved materials which is developed to describe the stress-strain behavior of caved materials by Salamon (1990):
\[
\varepsilon = \frac{\sigma_c}{E + \frac{\sigma_c}{E}}
\]
(19)

where \( \sigma_c \) is the uniaxial compressive strength of caved materials, and \( \varepsilon_m \) is the maximum possible strain of the caved materials.

The maximum possible strain \( (\varepsilon_m) \) merely depends on the bulking factor \( (b) \) and it can be calculated using the following equation (Yavuz, 2004):
\[
\varepsilon_m = \frac{b - 1}{b}
\]
(20)

The substitution of Eq. (20) into Eq. (19) yields
\[
\varepsilon = \frac{\sigma_c}{E + \frac{\sigma_c}{E}}
\]
(21)

When substituting Eq. (21) into Eq. (18), the stored elastic strain energy in the caved materials within the destressed zone can be obtained:
\[
U_E = \frac{E e^{-at} A_d H_d \sigma_c^2}{2 \left( E + 2 b \frac{\sigma_c}{E} \right)}
\]
(22)

A similar approach is used to calculate the viscoplastic strain energy \( (U_V) \). By substitution of the viscoplastic part of Eq. (14) into Eq. (16) as well as integration of it, we can have
\[
U_V = \left( \frac{t^2}{2K} - \frac{\lambda t^2}{2} \right) A_d H_d
\]
(23)

When substituting Eqs. (15) and (21) into Eq. (23), the final equation of stored viscoplastic strain energy in the caved materials within the destressed zone is acquired:
\[
U_V = \frac{A_d H_d \sigma_c^2}{2 \left( E + 2 b \frac{\sigma_c}{E} \right)} \left( E e^{-at} + \frac{\sigma_c^2}{E} - \lambda \right)
\]
(24)

In conclusion, the total stored strain energy in the caved materials within the destressed zone \( (U) \) can be obtained by replacing Eqs. (22) and (24) into Eq. (13) as follows:
\[
U = \frac{A_d H_d \sigma_c^2}{2 \left( E + 2 b \frac{\sigma_c}{E} \right)} \left( 2 E e^{-at} + \frac{\sigma_c^2}{E} - \lambda \right)
\]
(25)

4.1.3. Calculation of \( H_d \)

According to Eq. (1), the stored strain energy in the mined rock (coal) is equal to the stored strain energy in the caved materials within the destressed zone. Thus, the substitution of Eq. (12) and Eq. (25) into Eq. (1) leads to the equation for the height of destressed zone \( (H_d) \) in the long-term or time-dependent condition:
In Eq. (26), all units of force and distance are in N and m, respectively.

4.2. Total mining-induced stress

Calculation of the overburden load above a longwall panel that is not carried by the goaf but is transferred to the adjacent structures is very important to design of gateroad supports and pillar dimensions. Indeed, beyond the height of destressed zone, the overburden pressure will be transferred towards the front abutment, the adjacent access tunnels, the intervening barrier pillars as well as the panel rib-sides (Fig. 3). Thus, the total mining-induced stress \( (s_a) \) can be calculated as follows:

\[
\sigma_a = \sigma_v - \frac{(1+r)(1-2r)\gamma A_m s_v \left( \frac{H^2}{3} + H^2 + Hh_s \right)}{(E+\frac{H}{h_s})(2E \pi + \frac{E}{\pi} - \lambda)}
\]

(28)

The substitution of Eq. (26) into Eq. (27) leads to the following equation:

\[
\sigma_a = \sigma_v - \frac{g H_a}{(1+r)(1-2r)\gamma A_m s_v \left( \frac{H^2}{3} + H^2 + Hh_s \right)} \left( \frac{E \pi - \lambda}{E+\frac{H}{h_s}} \right)
\]

4.3. Abutment angle

As previously mentioned, it is important to calculate the load above the longwall panel that is not carried by the goaf but is transferred to the gates and pillars. This is referred to as the abutment load and it is typically calculated as the weight of material defined by an abutment angle, also referred to as the shear angle. In other words, this angle helps to easily quantify the overburden load above a goaf that is carried by adjacent gates and pillars known as the side abutment (see Fig. 4). It is suggested that the abutment angle correlates with the geological strength and bridging capacity of the overlying strata. However, no evidence has been collected to prove this assertion (Colwell et al., 1999). Some studies have recommended the use of a single abutment angle for all longwall panel designs (Mark, 1990; Colwell et al., 1999), while others suggested that it is variable and may depend on the overburden depth (Heasley, 2000). Generally, the abutment angle is determined by the extent of the destressed zone. On the other hand, the abutment angle can influence the distance that the mining-induced stress is transmitted into the neighboring abutments. Since the horizontal extension of destressed zone can be difficult to calculate theoretically, back analysis is utilized to determine the abutment angle in order to estimate the coefficient of stress concentration over gates.
and pillars in this research. Back-calculation of the abutment angle involves the estimated horizontal distance of the stress effect point from the rib-edge. Consequently, back-calculation is used to determine the abutment angle in which the horizontal distance is considered as the distance of gateroad or barrier pillar center from the panel edge. In fact, the horizontal distance does not necessarily determine the abutment angle in this research but helps to determine the vertical mining-induced stress and coefficient of stress concentration over adjacent gateroad and barrier pillar. It should be noted that a large number of corresponding abutment angles have been proposed from the estimated distances, i.e. 0.3H to 0.4H and 0.12H (Smart and Haley, 1987).

To back-calculate the abutment angle in this research, the situation (1) as shown in Fig. 3 (with relying on Fig. 4) is considered. Accordingly, the abutment angle is obtained as follows:

\[
\beta = \tan^{-1}\left(\frac{X}{H_d}\right) \tag{29}
\]

where X is the horizontal distance of the stress effect point from the panel edge.

4.4. Vertical induced stress

Determination of the vertical component of mining-induced stress is required for proper design of gate supports and pillar dimensions. To calculate this stress component, situation (2) as shown in Fig. 3 is considered (see also Fig. 4):

\[
\sigma_{a(V)} = \sigma_a \cos \beta \tag{30}
\]

where \(\sigma_{a(V)}\) is the vertical induced stress.

Substituting Eqs. (28) and (29) into Eq. (30), the vertical induced stress is obtained as follows:

\[
\sigma_{a(V)} = \sigma_v - \frac{(1+\nu)(1-2\nu)\nu^2A_m\sigma_v}{(1-\nu)^2} \frac{\left(\frac{H^2}{H_d^2} + Hh_s\right)}{E + \frac{H^2}{H_d^2}} \cos \left[\tan^{-1}\left(\frac{X}{H_d}\right)\right] \tag{31}
\]

According to trigonometric relations, we can write:

\[
\cos \left[\tan^{-1}\left(\frac{X}{H_d}\right)\right] = \frac{H_d^2}{H_d^2 + X^2} \tag{32}
\]

Substitution of Eq. (32) into Eq. (31) leads to the following equation:

\[
\sigma_{a(V)} = \sigma_v - \frac{(1+\nu)(1-2\nu)\nu^2A_m\sigma_v}{(1-\nu)^2} \frac{\left(\frac{H^2}{H_d^2} + Hh_s\right)}{E + \frac{H^2}{H_d^2}} \frac{H_d^2}{H_d^2 + X^2} \tag{33}
\]

4.5. Coefficient of stress concentration

To calculate the coefficient of stress concentration over gates and pillars, the secondary stress should be estimated. The secondary stress over the gates or pillars (\(\sigma_s\)) can be calculated by

\[
\sigma_s = \sigma_v + \sigma_{a(V)} \tag{34}
\]

Accordingly, the coefficient of stress concentration due to longwall mining is determined as follows:

\[
k = \frac{\sigma_s}{\sigma_v} \tag{35}
\]

Finally, substitution of Eqs. (33) and (34) into Eq. (35) yields the following equations:

\[
\sigma_v + \left[\sigma_v - \frac{(1+\nu)(1-2\nu)\nu^2A_m\sigma_v}{(1-\nu)^2} \frac{\left(\frac{H^2}{H_d^2} + Hh_s\right)}{E + \frac{H^2}{H_d^2}} \frac{H_d^2}{H_d^2 + X^2}\right] \sqrt{\frac{H^2}{H_d^2 + X^2}} \tag{36}
\]

or

\[
k = \frac{\sigma_v}{\sigma_v} \tag{37}
\]

5. Case study

In this research, Tabas coal mine is selected as the case study which is the largest and unique fully mechanized coal mine in Iran. This mine is located in the central part of Iran, 75 km south of Tabas in South Khorasan Province. The mine area is a part of Tabas-Kerman coalfield. Large volume of the coal reserve and appropriate geometry of the coal seams in Tabas have created suitable condition for application of longwall mining method. The Tabas coal deposit includes three minable seams including C1, B1 and B2. The C1 seam is the most important coal seam in Tabas and mined using mechanized longwall mining method. The thickness of coal seam ranges from 1.8 m to 2.2 m (averaging 2 m). The overburden depth varies from 100 m to 600 m. In this study, average value of 350 m is considered as the overburden depth in the modeling calculations. The width of longwall panels ranges from 200 m to 220 m (averaging 210 m) and their length is about 1000 m. The immediate roof rock above the panel is weak and contains 0.1–0.2 m thick mudstone and siltstone/sandstone interlayers. Also, sandstone channels in some areas within 3 m have potential water-bearing risk. The floor is about 1–1.3 m of weak seat-earth/mudstone, and there is a stronger mudstones and siltstones/sandstones layer under it (see Fig. 5). The average geometrical and geomechanical parameters of coal and overburden rocks of Tabas mechanized longwall mine are given in Table 3.

As a practical engineering example, the proposed model is utilized to calculate the coefficient of stress concentration over gate-road and pillar adjacent to the longwall panels of Tabas coal mine. The utilized constant values of experimental parameters in energy model calculations are considered according to Zhang et al. (2011), as shown in Table 4. In addition, the average bulking factor of roof rock strata (b) is considered to be 1.5 and the threshold value of stress (\(\sigma_t\)) is assumed to be 80% of uniaxial compressive strength of caved materials. The later assumption is considered based on the relation between yield point and peak strength point of stress-strain curve of rocks. Generally, threshold value of stress is the yield point of stress-strain curve which is commonly assumed to be about 80% of the peak compressive strength or stress (Hoek et al., 1997; Falakian, 2012). Since the maximum applied stress is equivalent to the maximum uniaxial compressive strength, the threshold...
input parameters on the output (coefficient of stress concentration) according to Eqs. (36) and (37). Accordingly, variations of the coefficient of stress concentration versus the maximum distance from the panel edge, abutment angle, height of destressed zone, extracted coal seam thickness, overburden depth, unit weight, elastic modulus, Poisson’s ratio, uniaxial compressive strength, bulking factor and pressure time of caved materials are shown in Fig. 6.

It can be concluded from Fig. 6a that the coefficient of stress concentration has an inverse relation with the distance from the panel edge. This result is in agreement with that of previous researches, such as Salamon (1964), Mark (1990) and Heasley (2000). As shown in Fig. 6b, the coefficient of stress concentration decreases with increase of the abutment angle. According to Fig. 6c, increasing the height of destressed zone generates a lower coefficient of stress concentration. At the same time, as the height of destressed zone increases to 350 m (equal to the overburden depth considered in this research), the stress field returns to its original status. Thus, the coefficient of stress concentration is equal to 1 and stress transfer to the neighboring structures will not accrue. It can be seen from Fig. 6d that with increasing the extracted coal seam thickness, the coefficient of stress concentration decreases. Indeed, the mining-induced stress decreases when the coal seam thickness increases. This result agrees with the results of investigations conducted by Majumder and Chakrabarty (1991) and Jayanthu et al. (2004). Fig. 6e indicates that with increasing depth of cover, the coefficient of stress concentration firstly increases and then decreases. For geometrical and geomechanical conditions of Tabas coal mine, the coefficient of stress concentration increases until the overburden depth reaches 180 m. After that, the coefficient of stress concentration decreases gradually. Accordingly, it can be concluded that relationship between the coefficient of stress concentration and the depth of cover is roughly parabolic. Fig. 6f and g demonstrates that the coefficient of stress concentration has the nonlinear inverse relation with both the rock mass unit weight and the elastic modulus. However, variations of the coefficient of stress concentration for higher values of the elastic modulus are negligible (Fig. 6g). Fig. 6h shows that the coefficient of stress concentration has a nonlinear relation with the Poison’s ratio of rock mass. According to Fig. 6i, the coefficient of stress concentration has close relation with the uniaxial compressive strength of caved materials, although variations are negligible for higher values of the uniaxial compressive strength. As seen from Fig. 6j, the coefficient of stress concentration increases with the increasing bulking factor of caved materials. However, variations are negligible for bulking factors greater than 1.3. As shown in Fig. 6k, the coefficient of stress concentration has a negative relation with the pressure time of caved materials. After the beginning of coal seam extraction and the occurrence of the first period of caving (for example, γ = 0.0001 year), the coefficient of stress concentration has the maximum possible value. When the pressure time of caved material increases, the coefficient of stress concentration decreases. For geometrical and geomechanical characteristics of the case study, the stress field returns to its original state after 2.75 years of the first caving.

According to Fig. 6, the coefficient of stress concentration obtained from the time-dependent energy model is in the range of 1–1.91 based on the actual data and experimental constants parameters. Accordingly, considering the mining-induced vertical stress, the maximum coefficient of stress concentration obtained from the proposed energy model is equal to 1.91.
formulas given in the literature. For this purpose, the coefficient of stress concentration is calculated using the equations of Salamon (1964), Majdi (1988) and Mark (1990), and compared with the result of energy model proposed in this research based on the actual data of Tabas coal mine (as shown in Table 3). The comparison results are given in Table 5. As can be seen, the result of proposed model is proven to be in a close agreement with the result of Majdi equation. In contrast, the Salamon and Mark equations show much higher values of coefficient of stress concentration than that of the energy model. It seems that these differences are induced due to the fact that the geomechanical characteristics of overburden rock mass are not considered in Salamon and Mark equations. As a result, the effect of strength properties is neglected in determining the mining-induced stress. Due to the complex condition of long-wall mining and existence of many effective parameters, judgment on the mining-induced stress only based on the geometrical parameters is rather far from the reality. For example, rock mass geomechanical characteristics and extracted seam thickness have considerable influences on the height of destressed zone, which itself affects the mining-induced stress and the coefficient of stress concentration. Therefore, it can be concluded that the proposed energy model and Majdi equation, which consider both the
geometrical and geomechanical parameters, are much closer to reality than Salamon and Mark equations.

It was mentioned before that the proposed time-dependent energy model predicts the coefficient of stress concentration in the range of 1–1.91 according to Fig. 6. As a general comparison, the results of four series of information collected from the literature (indicated in Table 2) and comparable analytical equations (shown in Table 5) as well as energy model results are summarized in Table 6. This comparison proved the close agreements that exist among the lower and upper limits of energy model and in situ measurements. Furthermore, the lower limits of empirical, analytical and numerical models are in agreement with the results of in situ measurement and proposed energy model. In contrast, the upper limits of these models are much greater than those of in situ and energy models. Generally, the main conclusion of this comparison is that the results of proposed energy model are in a close agreement with the results of in situ measurements as well as with the lower limit of other comparable models. Also, the general significance of the proposed analytical energy model compared with other analytical and empirical models is that it incorporates further effective parameters i.e., pressure time and strength properties in calculating the mining-induced stress. Since the proposed equation in this research is developed based on the combination of possible effective parameters, its results are close to the reality. Considering the above

Fig. 6. (continued).
8. Conclusions

A new time-dependent analytical model was developed in this study to calculate the longwall mining-induced stress and the coefficient of stress concentration. For this purpose, the height of the initial stressed zone was first determined based on the combination of the strain energy of the longwall mining and a rheological model of caved materials to determine the total mining-induced stress. Then, the vertically induced stress and the coefficient of stress concentration over gates and barrier pillars were estimated using the back-calculated abutment angle. The proposed model was used to determine the coefficient of stress concentration over gates and pillars of the Tabas longwall coal mine. Considering 1.5 years from the first caving in the main panel of this mine, the coefficients of stress concentration of 1.62 and 1.61 over the main gate and its adjacent pillar were obtained, respectively. Sensitivity analysis of the proposed model showed that Poisson's ratio, uniaxial compressive strength and bulking factor have significant influences on the coefficient of stress concentration. On the contrary, distance from the panel edge, abutment angle, height of stressed zone, extracted coal seam thickness, elastic modulus, unit weight and pressure time of the caved materials have negative influences on the coefficient of stress concentration. Unlike the two above categories, coefficient of stress concentration has a parabolic relationship with depth of cover. To assess the proposed model performance, its results were compared with the results of existing analytical, numerical and empirical models as well as with the results of in situ measurements. The comparative results confirm the agreement that exists between the proposed model and in situ measurements. Generally, the main significance of the proposed time-dependent energy model is that it incorporates the effects of geometrical and geomechanical parameters as well as pressure time of caved materials in estimating mining-induced stress. Finally, the key consequence of this study is that there is potential to determine the longwall mining-induced stress in long-term conditions. This facilitates the optimum design of gateroad supports and pillar dimensions. Therefore, safety and performance of these structures will be guaranteed during mining operation.

Conflict of interest

The authors wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

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Peng SS. Longwall mining, 2nd ed. Englewood, USA: Society for Mining, Metallurgy, and Exploration, Inc. (SME); 2006.


Table 5 Comparison of the results of existing analytical formulas with the proposed model to calculate the coefficient of stress concentration in Tabas coal mine.

<table>
<thead>
<tr>
<th>Analytical equations</th>
<th>Coefficient of stress concentration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salamon (1984)</td>
<td>3.35</td>
</tr>
<tr>
<td>Majdi (1988)</td>
<td>1.54</td>
</tr>
<tr>
<td>Mark (1990)</td>
<td>3.94</td>
</tr>
<tr>
<td>Time-dependent energy model</td>
<td>1.62</td>
</tr>
</tbody>
</table>

Table 6 Comparison of the results of proposed model with the results of existing methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>Coefficient of stress concentration</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1.08–1.85</td>
<td>Table 2</td>
</tr>
<tr>
<td>Empirical model</td>
<td>1.5–5</td>
<td>Tables 2 and 5</td>
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<tr>
<td>Analytical model</td>
<td>1.54–3.94</td>
<td></td>
</tr>
<tr>
<td>Numerical model</td>
<td>1.67–6</td>
<td>Table 2</td>
</tr>
<tr>
<td>Time-dependent energy model</td>
<td>1–1.91</td>
<td>Present paper</td>
</tr>
</tbody>
</table>


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