A simplified approach to directly consider intact rock anisotropy in Hoek–Brown failure criterion

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ARTICLE INFO

Article history:
Received 2 May 2014
Received in revised form 23 June 2014
Accepted 28 June 2014
Available online 7 August 2014

Keywords:
Anisotropy
Hoek–Brown failure criterion
Rock mechanics
Anisotropic parameter
Degree of anisotropy

ABSTRACT

Many rock types have naturally occurring inherent anisotropic planes, such as bedding planes, foliation, or flow structures. Such characteristic induces directional features and anisotropy in rocks’ strength and deformational properties. The Hoek–Brown (H–B) failure criterion is an empirical strength criterion widely applied to rock mechanics and engineering. A direct modification to H–B failure criterion to account for rock anisotropy is considered as the base of the research. Such modification introduced a new definition of the anisotropy as direct parameter named the anisotropic parameter ($K_b$). However, the computation of this parameter takes much experimental work and cannot be calculated in a simple way. The aim of this paper is to study the trend of the relation between the degree of anisotropy ($R_b$) and the minimum value of anisotropic parameter ($K_{min}$), and to predict the $K_{min}$ directly from the uniaxial compression tests instead of triaxial tests, and also to decrease the amount of experimental work.

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1. Introduction

The study of both intact rock and rock mass behaviors is a hot issue in the design of engineering applications on/in rocks. Rock mass behavior is usually assumed to be isotropic. According to Hudson and Harrison (2000), one of the engineering myths is that rocks are often supposed to be always safe under any engineering activities because of the assumptions of high strength and stiffness. The consequential difficulties which are faced at studying rocks are their inherent characteristics of discontinuity, anisotropy, inhomogeneity, and inelasticity.

Hence, the concept of design on/in rocks is that most of rock masses are under stresses and forces — regardless of the source of stresses — which influence the stability of the rock materials. These stresses and forces affect the rocks by inducing changes in the rock structures and forming discontinuities in rock masses. Discontinuity is a collective term used to include joints, fractures, bedding planes, rock cleavage, foliation, shear zones, faults, etc. The planes of anisotropy are also classified as discontinuities and fractures. These planes of anisotropy are known as inherent planes in rock material, e.g. pre-existing planes of weakness, bedding planes, or planes induced by the loading associated with construction or operation of the project, e.g. initiation and propagation of cracks (Bobet et al., 2009).

Inherent anisotropy is considered as the nature of rocks which affects the behavior and properties of rocks in particular for the metamorphic and sedimentary rocks. From rock mechanics point of view, anisotropic nature of rocks causes the difference of the rock strength with respect to the orientation of loading and inherent planes (Saroglou and Tsiambaos, 2008).

Hoek–Brown (H–B) failure criterion is one of the main criteria of rock failure mechanism and is used widely for design purposes in/on rock masses. This criterion is used to predict the rock strength with respect to the principal stresses ($\sigma_1, \sigma_3$). Most of modifications to H–B failure criterion for anisotropy are dealing with the anisotropic nature of rocks by modifying the rock constants ($m, s$) according to the angle between load orientation and inherent planes ($\beta$). These approaches are named indirect modifications to H–B failure criterion for anisotropy.

On the other hand, a reported modification to H–B failure criterion by Saroglou and Tsiambaos (2008) had considered the anisotropic nature of rocks directly in the original H–B failure criterion. Therefore, this approach is considered as direct modification to H–B failure criterion for anisotropy which defines a new parameter, the anisotropic parameter ($K_b$), and inserts it in the original equation of H–B failure criterion.

The main problem is how to define the trend and find out the relationship between the direct anisotropic parameter and rock strength classification proposed by Ramamurthy (1983). The strength anisotropy classification depends on the degree of anisotropy ($R_b$), which is the ratio of the maximum to the minimum
values of uniaxial compressive strength (UCS) of anisotropic intact rock samples. The lower the value of $R_c$, the closer the rock nature to be isotropic.

In order to compute the value of anisotropic parameter ($K_b$) at any given angle $\beta$, we need to make a triaxial compression test for the intact rock samples. On the contrary, evaluating the degree of anisotropy ($R_c$) for rock samples requires uniaxial compressive test for anisotropic intact rock samples. So, it is important to estimate the value of anisotropic parameter using uniaxial compression test rather than triaxial compression test. Consequently, a proposed relation between the direct anisotropic parameter and rock strength classification is introduced. The relationship between the anisotropic parameter and the UCS is analyzed.

2. Anisotropic nature of rocks

An anisotropic rock has different properties in different directions. This nature is governed by the special structure characteristics of rocks which are the uncontrollable nature of rocks: the minerals forming the rocks, the orientations of the minerals' crystals, and the interaction between different grains. This anisotropic nature of rocks can be stated, according to Bagheripour et al. (2011), as follows:

(1) Most foliated metamorphic rocks, such as schist, slates, gneisses and phyllites, contain a natural orientation in their flat/long minerals or a banding phenomenon which results in anisotropy in their mechanical properties.

(2) Stratified sedimentary rocks like sandstone, shale or sandstone—shale alteration often display anisotropic behaviors due to presence of bedding planes.

(3) Anisotropy can also be exhibited by igneous rocks having flow structures as may be observed in rhyolites (Matsukura et al., 2002).

These anisotropies are often referred to as inherent anisotropy, and the corresponding rocks are sometimes categorized as intact anisotropic rock.

The strength anisotropy definition means that the directional characteristics of rocks, such as deformability modulus, strength, brittleness, permeability and discontinuity frequency, are the function of angle between load orientation and inherent planes of anisotropy, $\beta$ (Fig. 1).

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Previous studies tried to solve the problem of representation of anisotropy in rock strength criterion. Attempts of classification of the rocks were made based on the anisotropic properties (Tsidi, 1990; Ramamurthy, 1993); these classifications are deterministic for the degree of anisotropy in the examined rocks. These studies began with Jaeger definition (Jaeger, 1960), followed by Donath experimental work (Donath, 1964) which dealt with the shear failure and behavior in anisotropic rocks, and the Walsh–Brace theory (Walsh and Brace, 1964) of fracture criterion for brittle anisotropic rocks. Hoek (1964) proposed a definition of the failure and fracture in the anisotropic rocks based on the “single weakness plane of failure” theory of Jaeger. In addition to these efforts, McLamore and Gray (1967) studied the mechanical behavior of the anisotropic sedimentary rocks. Similarly, Attewell and Sandford (1974) contributed to this field by preparing an experimental and mechanical study for the shear strength of the anisotropic brittle rocks.

Hoek and Brown (1980) proposed a modification to the failure criterion considering rock anisotropy. Colak and Unlu (2004) presented the variations of rock parameter $m_i$ for the intact transverse anisotropic rock in $H$–$B$ failure criterion. Last but not least, there was a direct modification which was discussed by Saroglou and Tsiambaos (2008), where a direct modification was added to the original $H$–$B$ failure criterion to take the anisotropy into account.

3. Hoek–Brown failure criterion

The $H$–$B$ failure criterion (Hoek et al., 2002) was developed using empirical data and considered the relation between the principal stresses. This relation is governed by three parameters ($m$, $s$, $\sigma_3$) which are the material constants and the compressive strength of rock material, respectively. Constants $s$ and $m$ define structural pattern, or quality, of rock mass and rock type, respectively. The constant $s$ predominantly affects the “tensile” strength of the rock and its influence is the most important at very low confinement ($\sigma_3$).

The original criterion depends on the compressive strength $\sigma_{ci}$ and the material constants $m$ and $s$. For inhomogeneous and anisotropic rocks, it is not easy to determine the rock strength which is very important for the applications of engineering in/on rocks (civil and mining). The empirical failure criterion initially proposed by Hoek and Brown for intact rock is described as

$$
\sigma_1 = \sigma_3 + \sigma_{ci} \left( m \frac{\sigma_3}{\sigma_{ci}} + s \right) ^ \alpha
$$

(1)

where $\sigma_1$ and $\sigma_3$ are the major and minor principal stresses, respectively; and $\alpha$ is the material constant. In case that the rock material is intact, the equation of the $H$–$B$ failure criterion for intact rock can be rewritten as

$$
\sigma_1 = \sigma_3 + \sigma_{ci} \left( m \frac{\sigma_3}{\sigma_{ci}} + 1 \right) ^ {0.5}
$$

(2)

where $m$ is equal to the constant $m_i$ for intact rock, $\alpha = 0.5$ and $s = 1$.

3.1. Original Hoek–Brown failure criterion

The determined values of the $H$–$B$ criterion parameters for intact rock ($\sigma_{ci}$, $m_i$, $s$) are defined based on the results of uniaxial and triaxial tests, when loading is applied perpendicularly to the planes of anisotropy (foliation or bedding). The values of $\sigma_{ci}$ and $m_i$ will be significantly different in case that failure will occur in the direction of such a plane (Colak and Unlu, 2004).
Hoek and Brown (1980) considered that the “single plane of weakness” theory (Jaeger, 1960) is sufficient for the prediction of strength, when the rock behaves in anisotropy way due to the presence of a single plane of weakness (e.g. discontinuity plane), but did not describe adequately the strength behavior of intact rock possessing inherent anisotropy, due to the presence of bedding or foliation, as Ramamurthy (1993) stated, as in the case of siltstones, schists, gneisses, etc.

Consequently, in order to predict the strength of intact anisotropic rock, Hoek and Brown (1980) suggested that the value of the constants \( m \) and \( s \) of their empirical criterion should be altered accordingly based on the orientation of the foliation plane relative to the principal loading axis, \( \beta \). Rather than following the approach proposed by McLamore and Gray (1967), Hoek and Brown adopted a series of empirical equations to modify the material constants \( m \) and \( s \) which have been used in the original formulation. The resulting equations for \( m \) and \( s \) are

\[
m = m_1 \left( 1 - A e^{-\theta^2} \right) \tag{3}
\]

\[
s = 1 - P e^{-\zeta^2} \tag{4}
\]

where \( m_1 \) is the value of \( m \) for the intact rock, \( A \) and \( P \) are constants, and \( \theta \) and \( \zeta \) are expressed as

\[
\theta = \frac{\beta - \zeta_m}{A_2 + A_3 \beta} \tag{5}
\]

\[
\zeta = \frac{\beta - \zeta_s}{P_2 + P_3 \beta} \tag{6}
\]

where \( \zeta_m \) is the value of \( \beta \) at minimum \( m \), \( \zeta_s \) is the value of \( \beta \) at minimum \( s \), and \( A_2, A_3, P_2 \) and \( P_3 \) are constants. The values of \( m \) and \( s \) for different values of the discontinuity angle \( \beta \) are calculated by means of the linear regression analysis (Hoek and Brown, 1980) in the underground excavation in rocks. The variations of the constants \( m \) and \( s \) relative to the orientation of the joints are given by

\[
\frac{m}{m_1} = 1 - A \exp \left[ - \left( \frac{\beta - \zeta_m}{A_2 + A_3 \beta} \right)^4 \right] \tag{7}
\]

\[
s = 1 - P \exp \left[ - \left( \frac{\beta - \zeta_s}{P_2 + P_3 \beta} \right)^4 \right] \tag{8}
\]

where constants \( A \) and \( P \) can be computed by

\[
A = m_1 - \frac{m_{\min}}{m_1} \tag{9}
\]

\[
P = 1 - s_{\min} \tag{10}
\]

The variation of the values \( m/m_1 \) and \( s \) relative to the orientation angle, \( \beta \), for Martinsburg slate (Donath, 1964) are presented in Fig. 2, in which the values of \( s \) at \( \beta = 30^\circ \) and \( 45^\circ \) are also given. The negative value of \( s \) obtained in such cases has no physical significance and is set to zero in order to avoid mathematical complications in the subsequent analysis (Hoek and Brown, 1980).

3.2. Direct modification to Hoek--Brown failure criterion

Saroglou and Tsiambaos (2008) found that it is obvious that the original failure criterion modifications for the anisotropic rock material depend on the trial and error (the back analysis), besides the large number of experiments which may be not provided for each case. Also there are no modifications about natural (inherent) anisotropy such as schistosity, foliation and bedding planes.

The modification to H--B failure criterion has been carried out by incorporation of a new parameter, called “anisotropic parameter” \( K_\beta \), representing the effect of strength anisotropy:

\[
\sigma_1 = \sigma_3 + \sigma_{c\beta} \left( K_\beta \frac{m_{\min} \sigma_3}{\sigma_{c\beta}} + 1 \right)^{0.5} \tag{11}
\]

where \( \sigma_{c\beta} \) is the UCS at an angle of loading \( \beta \). The direct modification to the H--B failure criterion, which is considered as an experimental model (Fig. 3) in order to account for the effect of anisotropy on strength, is based on the inclusion of directional properties (Saroglou and Tsiambaos, 2008). The modification is

![Diagram](image326x479 to 529x727)
based on (1) the variation of the UCS of intact rock due to presence of foliation ($\sigma_{fj}$); (2) the anisotropic strength parameter $K_b$ that denotes the range between the minimum and the maximum strengths of intact anisotropic rock; and (3) the constant $s$ is equal to the unity ($=1$) due to the intact state of rock (Hoek et al., 2002).

The minimum value of $K_b$, $K_{min} = K_{30}, 45$, occurs at angle $\beta_{min}$. This angle $\beta_{min}$ between the axis of major principal stress ($\sigma_1$) and the foliation planes ranges from 30° to 45° (Donath, 1966).

The function of the reported anisotropic parameter is not only to decrease the anisotropic behavior of the intact rock in the H–B criterion but also to describe the degree of anisotropy of the tested intact rock. $K_b$, a parameter added, is computed and processed to study its variation with the degree of anisotropy. The anisotropic parameters of Penrhyn slate, gneiss A, gneiss B, schist as well as marble are listed in Table 1 (Saroglou and Tsiambaos, 2008).

Although this modification is introduced in the failure criterion, there still need a large number of experiments for calculation of $K_b$. Also, the modification of Saroglou et al. is dedicated to the metamorphic rocks neglecting sedimentary rocks. There is no linkage between the descriptive analyses of anisotropy and the anisotropic parameter $K_b$.

3.3. The variation of the anisotropic parameter vs. the anisotropic strength

Ramamurthy (1993) had given a classification of strength anisotropy based on the ratio $R_c$, which describes the degree of anisotropy using a scalar quantity. Using degree of anisotropy, it is possible to calculate the strength for each anisotropic rock type directly based on the anisotropic parameter $K_b$ and the constant $m_i$ depending on the type of rock and the texture (Saroglou and Tsiambaos, 2008).

It is evident that the ratio $K_{90}/K_{min}$ is greater for the rocks with a high degree of anisotropy, e.g. Penrhyn slate, gneiss, and reduces significantly for the rocks with a low degree of anisotropy, e.g. schist and marble, where $K_{90}$ is the value of the parameter $K_b$ when loading is perpendicular to the foliation, equal to unity ($=1$), and $K_{min}$ is its value at the orientation of minimum strength at $\beta = 30° - 45°$.

Both the anisotropic parameter $K_b$ and the strength anisotropy depend on the angle of loading relative to the planes of the inherent anisotropy (Table 2). As $R_c$ is the ratio of the compressive strength at $\beta = 90°$ to the minimum value of compressive strength, the variation between the parameter $K_b$ and anisotropic strength should be compared.

4. Simplified approach to consider rock anisotropy in Hoek–Brown criterion

The methodology would be arranged the same as that referred by Saroglou and Tsiambaos (2008). The procedure of $K_b$ calculations is going to be applied to data obtained from confining compressive test on other 10 anisotropic intact rock samples. These 10 rock samples are Martinsburg slate (Donath, 1964), South African slate (Hoek, 1964), Green River shale II (McLamore and Gray, 1967), fractured sandstone (Horino and Ellickson, 1970), model rock (weak sandstone) (Bagheripour and Mostyn, 1996), sandstone and siltstone ( Yaşar, 2001), model rock (Bagheripour et al., 2011), slates S and Z (Saéidi et al., 2013).

According to Fig. 4, the paper’s main focus is to plot the relation between the calculated anisotropic strength of tested anisotropic intact rocks and the minimum value of the anisotropic parameter $K_{min}$. This methodology aims at linking the anisotropic strength classification introduced by Ramamurthy (1993) and the associated anisotropic parameter. The proposed relation would be used directly once the anisotropic strength is known or computed; also the authors show the compatibility between the anisotropic strength $R_c$ and the ratio $K_{90}/K_{min}$. Mathematically, the proposed relation is introduced to generally predict the minimum value of anisotropic parameter $K_{min}$ given that the value of anisotropic strength $R_c$ is obtained. It can be represented by

$$K_{min} = F(R_c)$$

The results would be the values of the anisotropic parameter for each orientation; these results are extracted from the tested rock samples. There would be a linear relation between the anisotropic strength $R_c$ and the ratio of $K_{90}/K_{min}$ for the tested rock samples. It can be noted that the relation would satisfy the solution for finding the value of $K_{min}$ for any given anisotropic rock, once the anisotropic strength is given or calculated. Finally, it is logic that the relationship between $K_{min}$ and $1/R_c$ is in direct proportion.

4.1. Limitations of proposed relation

The limitations of the proposed relation are described as follows:

1. The relationship shows continuity within the ranges of the anisotropic strength classification proposed by Ramamurthy (1993).
2. For the isotropic rock ($R_c = 1$), the value of $K_{min}$ may equal the unity ($=1$), according to the anisotropic strength classification proposed by Ramamurthy (1993).
3. The value of $K_{min}$ cannot be negative or zero under any circumstances, according to Saroglou and Tsiambaos (2008).

<table>
<thead>
<tr>
<th>Rock type</th>
<th>$m_i$</th>
<th>Anisotropic parameter $K_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\beta = 0°$</td>
</tr>
<tr>
<td>Penrhyn slate</td>
<td>–</td>
<td>0.89</td>
</tr>
<tr>
<td>Gneiss A</td>
<td>24.6</td>
<td>1.79</td>
</tr>
<tr>
<td>Gneiss B</td>
<td>23.2</td>
<td>0.88</td>
</tr>
<tr>
<td>Schist</td>
<td>9.5</td>
<td>0.96</td>
</tr>
<tr>
<td>Marble</td>
<td>9.6</td>
<td>0.99</td>
</tr>
</tbody>
</table>

### Table 2

<table>
<thead>
<tr>
<th>Rock type</th>
<th>$K_{90}/K_{min}$</th>
<th>$R_c$</th>
<th>Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penrhyn slate</td>
<td>4</td>
<td>3.3</td>
<td>Medium anisotropy</td>
</tr>
<tr>
<td>Gneiss A</td>
<td>2.3</td>
<td>2.2</td>
<td>Medium anisotropy</td>
</tr>
<tr>
<td>Gneiss B</td>
<td>2.5</td>
<td>3.8</td>
<td>Medium anisotropy</td>
</tr>
<tr>
<td>Schist</td>
<td>1.33</td>
<td>1.25</td>
<td>Low anisotropy</td>
</tr>
<tr>
<td>Marble</td>
<td>1.09</td>
<td>1.14</td>
<td>Isotropy to low anisotropy</td>
</tr>
</tbody>
</table>
4.2. Processing of the results

The gathered data can be listed in Table 3 for deduction of the relationship between the anisotropic strength and the minimum value of the anisotropic parameter.

### 4.2.1. Notes of the gathered data

The notes of the gathered data are described as follows:

1. The minimum value of the UCS is usually obtained at \( \beta = 30^\circ\), except in case of schist (at \( \beta = 45^\circ\)).
2. The minimum value of the anisotropic parameter (\( K_{\text{min}} \)) is usually obtained at \( \beta = 30^\circ\), except in cases of gneiss A (at \( \beta = 45^\circ\)), Green River shale II (at \( \beta = 45^\circ\)), and slate S (at \( \beta = 60^\circ\)).
3. The maximum value of the UCS is mostly obtained at \( \beta = 90^\circ\), except in cases of Green River shale II (at \( \beta = 90^\circ\)) and South African slate (at \( \beta = 0^\circ\)).
4. The studied samples of anisotropic rocks include 5 low anisotropic rocks, 4 medium anisotropic rocks, 4 high anisotropic rocks, and 2 very high anisotropic rocks, according to the anisotropic strength classification (Ramamurthy, 1993).

### Table 3

<table>
<thead>
<tr>
<th>Rock code</th>
<th>Rock type</th>
<th>( \sigma_{90} ) (MPa)</th>
<th>( \sigma_{90} ) (MPa)</th>
<th>( \beta ) for ( \sigma_{90} ) (°)</th>
<th>( \sigma_{90} ) (MPa)</th>
<th>( \sigma_{90} ) (MPa)</th>
<th>Rock class</th>
<th>( K_{\text{min}} )</th>
<th>( \beta ) for ( K_{\text{min}} ) (°)</th>
<th>( K_{0}/K_{\text{min}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Martinsburg slate</td>
<td>155</td>
<td>18</td>
<td>30</td>
<td>92.3</td>
<td>14.08</td>
<td>8.61</td>
<td>Very high anisotropy</td>
<td>0.15</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>Penrhyn slate</td>
<td>206.6</td>
<td>38.2</td>
<td>30</td>
<td>159.7</td>
<td>6.23</td>
<td>4.85</td>
<td>High anisotropy</td>
<td>0.25</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>Sandstone</td>
<td>64</td>
<td>40.67</td>
<td>30</td>
<td>53</td>
<td>4.35</td>
<td>1.45</td>
<td>Low anisotropy</td>
<td>0.756</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>Siltstone</td>
<td>85</td>
<td>66.1</td>
<td>20</td>
<td>76</td>
<td>6</td>
<td>1.23</td>
<td>Low anisotropy</td>
<td>0.663</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>Gneiss A</td>
<td>66.5</td>
<td>35.5</td>
<td>30</td>
<td>39.4</td>
<td>24.6</td>
<td>2.2</td>
<td>Medium anisotropy</td>
<td>0.38</td>
<td>45</td>
</tr>
<tr>
<td>6</td>
<td>Gneiss B</td>
<td>85.7</td>
<td>23.4</td>
<td>30</td>
<td>45.4</td>
<td>23.2</td>
<td>3.8</td>
<td>Medium anisotropy</td>
<td>0.41</td>
<td>30</td>
</tr>
<tr>
<td>7</td>
<td>Schist</td>
<td>66.4</td>
<td>58.2</td>
<td>45</td>
<td>64.3</td>
<td>9.5</td>
<td>1.25</td>
<td>Low anisotropy</td>
<td>0.75</td>
<td>30</td>
</tr>
<tr>
<td>8</td>
<td>Marble</td>
<td>88.4</td>
<td>76.1</td>
<td>30</td>
<td>88.1</td>
<td>9.6</td>
<td>1.14</td>
<td>Low anisotropy</td>
<td>0.91</td>
<td>30</td>
</tr>
<tr>
<td>9</td>
<td>Green River shale</td>
<td>106.6</td>
<td>71.66</td>
<td>30</td>
<td>127.7</td>
<td>4.33</td>
<td>1.48</td>
<td>Low anisotropy</td>
<td>0.55</td>
<td>45</td>
</tr>
<tr>
<td>10</td>
<td>South African slate</td>
<td>106.65</td>
<td>29</td>
<td>30</td>
<td>125</td>
<td>7.96</td>
<td>3.68</td>
<td>Medium anisotropy</td>
<td>0.34</td>
<td>30</td>
</tr>
<tr>
<td>11</td>
<td>Fractured sandstone</td>
<td>183.1</td>
<td>35.5</td>
<td>30</td>
<td>120.7</td>
<td>22.87</td>
<td>5.16</td>
<td>High anisotropy</td>
<td>0.247</td>
<td>30</td>
</tr>
<tr>
<td>12</td>
<td>Model rock</td>
<td>13.6</td>
<td>1.1</td>
<td>30</td>
<td>7.4</td>
<td>14.81</td>
<td>12.36</td>
<td>Very high anisotropy</td>
<td>0.11</td>
<td>30</td>
</tr>
<tr>
<td>13</td>
<td>Model rock (weak sandstone)</td>
<td>42.2</td>
<td>8</td>
<td>30</td>
<td>28</td>
<td>13.5</td>
<td>5.275</td>
<td>High anisotropy</td>
<td>0.253</td>
<td>30</td>
</tr>
<tr>
<td>14</td>
<td>Slate S</td>
<td>65</td>
<td>15</td>
<td>30</td>
<td>50</td>
<td>–</td>
<td>4.33</td>
<td>High anisotropy</td>
<td>0.25</td>
<td>60</td>
</tr>
<tr>
<td>15</td>
<td>Slate Z</td>
<td>92</td>
<td>11</td>
<td>30</td>
<td>32</td>
<td>–</td>
<td>3.06</td>
<td>Medium anisotropy</td>
<td>0.33</td>
<td>30</td>
</tr>
</tbody>
</table>

Note: \( \sigma_{90} \) with subscripts 0, 90 and min represents UCSs at \( \beta = 0^\circ\), \( 90^\circ\) and \( \beta_{\text{min}} \) respectively. \( m \) with a subscript 90 means that the value of \( m \) is determined by the fitting procedure from a series of triaxial tests perpendicular to the planes of anisotropy at \( \beta = 90^\circ\).

### Table 4

The expected ranges of \( K_{\text{min}} \) using \( R_c \) classification.

<table>
<thead>
<tr>
<th>( R_c )</th>
<th>Class</th>
<th>( K_{\text{min}} )</th>
<th>Legend</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 &lt; ( R_c ) ≤ 1.1</td>
<td>Isotropic rock</td>
<td>0.85 ≤ ( K_{\text{min}} ) &lt; 0.91</td>
<td></td>
</tr>
<tr>
<td>1.1 &lt; ( R_c ) ≤ 2</td>
<td>Low anisotropy</td>
<td>0.53 ≤ ( K_{\text{min}} ) &lt; 0.85</td>
<td></td>
</tr>
<tr>
<td>2 &lt; ( R_c ) ≤ 4</td>
<td>Medium anisotropy</td>
<td>0.31 ≤ ( K_{\text{min}} ) &lt; 0.53</td>
<td></td>
</tr>
<tr>
<td>4 &lt; ( R_c ) ≤ 6</td>
<td>High anisotropy</td>
<td>0.23 ≤ ( K_{\text{min}} ) &lt; 0.31</td>
<td></td>
</tr>
<tr>
<td>( R_c ) &gt; 6</td>
<td>Very high anisotropy</td>
<td>( K_{\text{min}} ) &lt; 0.23</td>
<td></td>
</tr>
</tbody>
</table>

### 4.2.2. Regression and curve fitting for the outputs

The linear formulation resulting from the output data in Table 4 is represented in Fig. 5, and can be expressed by

\[
K_{0}/K_{\text{min}} = 0.694R_c + 0.53
\]  

(13)

The coefficient of determination \( R^2 \) for Eq. (13) is 0.9826. Also as is indicated by Eq. (13), the variables \( R_c \) and \( K_{0}/K_{\text{min}} \) present a direct proportion. In case of perfect isotropic samples (\( R_c = 1 \)), the value \( K_{0}/K_{\text{min}} \) is 0.694 × 1 + 0.53 = 1.224.

Accordingly, the practical maximum value of ratio \( K_{0}/K_{\text{min}} \) shows that the value for perfectly isotropic intact rock sample is not equal to 1 as it is assumed previously.

Also a linear regression is carried out in order to evaluate the best fit of the proposed relation between \( R_c \) and \( K_{\text{min}} \) for the studied rock samples. The linear formulation is represented as a power function as shown in Fig. 6, and can be written as

\[
K_{\text{min}} = 0.9R_c^{-0.79}
\]  

(14)

The coefficient of determination \( R^2 \) for Eq. (14) is 0.9538. Also as indicated by Eq. (14), there is an inverse proportion between the two variables \( R_c \) and \( K_{\text{min}} \). Also the maximum value of \( K_{\text{min}} \) can be obtained when the value of \( R_c \) is maximum, in case of perfectly isotropic samples (\( R_c = 1 \)), and the value \( K_{\text{min}} \) is \( 0.9 \times 1^{-0.79} = 0.9 \).
Finally, it shows that the maximum value of $K_{\min}$ for perfect isotropic rocks is not the unity ($\frac{1}{\sigma_1}$) as it is assumed previously.

Thus, a modification to the anisotropic strength classification can be conducted using this relation to obtain the range of $K_{\min}$ values according to the obtained ranges of $R_c$. This relation is subjected to continuous modification in terms of extra experimental data, as shown in Table 4 and Fig. 7.

On the other hand, it is depicted from Table 5 and Fig. 8 that the values of $K_{\min}$ obtained by the proposed relation agree well with the computed values obtained from the tested anisotropic rock samples. Nevertheless, there is a slight deviation in the expected values of $K_{\min}$ than the computed values especially for the range of $K_{\min}$ larger than 0.4 as it is noticed in Fig. 8.

Furthermore, there is a close relation between the computed and the expected values of $K_{\min}$ for high to very high anisotropy rock samples ($K_{\min} < 0.4$). Thus, the proposed relation shows a good ability in prediction of the values of $K_{\min}$ using the UCS ($R_c$) rather than the excessive triaxial testing to extract the value of $K_{\min}$ from tested rock samples.

5. Conclusions

It is obvious that the anisotropy characteristics of rocks have effects on their mechanical properties, which depend on the load orientation relative to the inherent anisotropic planes. While the main role of the anisotropy, from rock mechanics point of view, is represented in the differentiation of the rock strength associated with the orientation of loading and inherent planes. Also, the strength anisotropy definition means that the directional characteristics of the rocks, such as deformability modulus, strength, brittleness, permeability and discontinuity frequency, are functions of the angle between load orientation and inherent planes of anisotropy, $\beta$.

The indirect modification to H–B failure criterion aims at altering the values of the rock parameters, depending on a large number of experiments which may be not provided for each case. On the other hand, the direct modification to H–B failure criterion to account for the anisotropy is the insertion of the anisotropic parameter into the criterion. The authors use the terminology, the anisotropic strength parameter $K_b$ which presents the range between the minimum and the maximum strengths of the intact anisotropic rock. The minimum value of the anisotropic strength parameter, $K_{\min}$, can be obtained when loading is performed at the angle $\beta_{\min}$. This angle “$\beta_{\min}$” is defined as the angle between the major principal stress ($\sigma_1$) and the foliation planes, ranging between $30^\circ$ and $45^\circ$.

The proposed relation between the minimum value of the anisotropic parameter $K_{\min}$ and the anisotropic strength classification $R_c$, is obtained through conducting the experimental work on

<table>
<thead>
<tr>
<th>Rock code</th>
<th>Rock type</th>
<th>$R_c$</th>
<th>Measured $K_{\min}$</th>
<th>Computed $K_{\min}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Martinsburg slate</td>
<td>8.61</td>
<td>0.15</td>
<td>0.166</td>
</tr>
<tr>
<td>2</td>
<td>Penrhyn slate</td>
<td>4.85</td>
<td>0.25</td>
<td>0.26</td>
</tr>
<tr>
<td>3</td>
<td>Sandstone</td>
<td>1.45</td>
<td>0.756</td>
<td>0.68</td>
</tr>
<tr>
<td>4</td>
<td>Siltstone</td>
<td>1.23</td>
<td>0.063</td>
<td>0.77</td>
</tr>
<tr>
<td>5</td>
<td>Gneiss A</td>
<td>2.2</td>
<td>0.38</td>
<td>0.48</td>
</tr>
<tr>
<td>6</td>
<td>Gneiss B</td>
<td>3.8</td>
<td>0.41</td>
<td>0.316</td>
</tr>
<tr>
<td>7</td>
<td>Schist</td>
<td>1.25</td>
<td>0.75</td>
<td>0.762</td>
</tr>
<tr>
<td>8</td>
<td>Marble</td>
<td>1.14</td>
<td>0.91</td>
<td>0.83</td>
</tr>
<tr>
<td>9</td>
<td>Green River shale II</td>
<td>1.48</td>
<td>0.55</td>
<td>0.67</td>
</tr>
<tr>
<td>10</td>
<td>South African slate</td>
<td>3.68</td>
<td>0.34</td>
<td>0.325</td>
</tr>
<tr>
<td>11</td>
<td>Fractured sandstone</td>
<td>5.16</td>
<td>0.247</td>
<td>0.249</td>
</tr>
<tr>
<td>12</td>
<td>Model rock (weak sandstone)</td>
<td>12.36</td>
<td>0.11</td>
<td>0.125</td>
</tr>
<tr>
<td>13</td>
<td>Model rock</td>
<td>5.275</td>
<td>0.253</td>
<td>0.245</td>
</tr>
<tr>
<td>14</td>
<td>Slate S</td>
<td>4.33</td>
<td>0.25</td>
<td>0.286</td>
</tr>
<tr>
<td>15</td>
<td>Slate Z</td>
<td>3.06</td>
<td>0.33</td>
<td>0.376</td>
</tr>
</tbody>
</table>
15 anisotropic rock samples of low to very high anisotropy. It is found that there is a good match between the expected values of $K_{\text{min}}$ using the proposed relation and the computed values of $K_{\text{min}}$ for the same tested rock samples. Also, for low and medium anisotropy rock samples, further investigations could be helpful to reveal the significant differences in values of $K_{\text{min}}$.

Conflict of interest

The authors wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

Acknowledgments

We are grateful for the helpful discussions with Prof. Hany M. Helal (Former Higher Education Minister, Egypt) and also for his useful comments.

References


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