Full length article

Theoretical and numerical studies of crack initiation and propagation in rock masses under freezing pressure and far-field stress

Yongshui Kang \textsuperscript{a,}\textsuperscript{*}, Quansheng Liu \textsuperscript{a}, Xiaoyan Liu \textsuperscript{b}, Shibing Huang \textsuperscript{a}

\textsuperscript{a} State Key Laboratory of Geomechanics and Geotechnical Engineering, Institute of Rock and Soil Mechanics, Chinese Academy of Sciences, Wuhan, 430071, China
\textsuperscript{b} Key Laboratory of Rock Mechanics in Hydraulic Structural Engineering of Ministry of Education, School of Civil Engineering and Architecture, Wuhan University, Wuhan, 430072, China

1. Introduction

Low temperatures and harsh weather can be observed in nearly half of the earth's land area in winter. Frost weathering in cold regions, where a quarter of the land mass is under permafrost or covered by snow, poses a serious threat to the stability of geotechnical engineering endeavors. Frost shattering caused by freeze–thaw cycles in moist rocks, as well as expansion and contraction due to temperature changes, is the major reason for physical weathering of rocks (Yang and Chen, 2004; Seto, 2010; Kang et al., 2012a, 2013). The freeze–thaw deterioration of rock is mainly reflected by the propagation of cracks under freezing pressure. Therefore, the water–ice phase transition and its effect on the evolution of the crack network should be investigated to reveal the mechanism of freeze–thaw damage to rocks.

The freeze–thaw action is caused by temperature fluctuations around the freezing point of water. Freeze–thaw action, which is also referred to as ice crystal growth or frost shattering, occurs when water in cracked rock is frozen and expands. As shown in Fig. 1, when water in cracks is frozen, a volume expansion of approximately 9% occurs, initiating a large freezing pressure that deepens and expands the crack width. When ice in the rock thaws, the water can flow further into the rock through these new cracks, facilitating the next freeze–thaw cycles. Thus, the repeated freeze–thaw processes of water in the cracks have long been recognized as a significant incentive for mechanical weathering of rocks (Hilbich, 2010; Su et al., 2010).

Frost damage in rocks is mainly caused by crack propagation under freezing pressure. The heterogeneous features of natural rocks give rise to various types of defects or local stress concentrations that might be the sources of microcracks. Based on the assumption that pre-existing microcracks are considered the most likely sources for crack propagation, Paterson and Wong (2005) analyzed the sliding crack model in detail.

In recent years, many studies have concentrated on the frost damage induced by freeze–thaw action in rock masses. However, as a result of the complicated freeze–thaw process and mathematical difficulties, most of the previous research concentrated on...
fracture toughness) at which point a wing crack is nucleated. In this regard, the present paper tries to present a theoretical method to predict crack propagation under freezing pressures and far-field stresses. The direction and length of crack propagation are derived based on the Griffith theory of brittle fracture mechanics. The conditions necessary for different coalescence modes of cracks are also studied.

2. Stress intensity factors of freezing-induced rock cracks

2.1. Stress intensity factors

Rock is basically regarded as a brittle material and as such it cannot resist large tensile stresses. In general, cracks in rocks under freezing pressures and far-field stresses can be regarded as a mixed mode of tensile crack (Mode I) and shear crack (Mode II). The freezing pressure caused by volume expansion of ice can be considered as a normal pressure imposed on the crack plane, while the far-field stress might induce both normal and shear stresses on the crack plane. The sliding crack model considers the source of tensile stress concentrations located at the tips of inclined pre-existing cracks. The far-field stress would induce shear traction on the crack plane. If this shear traction is sufficiently high to overcome the frictional resistance along the interface between the crack and the ice, frictional slip occurs and a tensile stress concentration is induced at the tips of the sliding crack, with the appearance of wing cracks (Matsuoka, 2008). The driving force for nucleation is characterized by the stress intensity factor (SIF), \( K_i \), at the position of wing crack initiation. With the increasing load or freezing pressure, \( K_i \) will increase to the critical value \( K_{IC} \) (the fracture toughness) at which point a wing crack is nucleated.

Numerically, the freezing pressure on the crack might not be uniform; therefore attention should be focused on the SIF induced by non-uniform stress. As shown in Fig. 2a, the model is conceived as a plane stress problem. Normal and shear stresses are non-uniformly distributed on the faces of the crack. Therefore, the integration method should be used for this problem. The non-uniform stress is roughly transformed into numerous point forces as shown in Fig. 2b, which has been studied in previous research.

The SIFs at a crack tip caused by the point force on the crack plane can be expressed as follows (Yin, 1992; China Aviation Academy, 1993):

\[
\begin{align}
\Delta K_I &= \int_{-a}^{+a} \frac{P(x)dx}{2\sqrt{\pi a}} \sqrt{\frac{a+x}{a-x}} + \frac{\tau(x)dx}{2\sqrt{\pi a}} \left( \frac{k - 1}{k + 1} \right) \\
\Delta K_{II} &= \int_{-a}^{+a} \left[ -\frac{P(x)}{2\sqrt{\pi a}} \left( \frac{k - 1}{k + 1} \right) + \frac{\tau(x)}{2\sqrt{\pi a}} \left( \frac{a+x}{a-x} \right) \right] dx
\end{align}
\]

where \( P(x) \) and \( \tau(x) \) are the normal and shear forces at point \( x \), respectively; \( a \) is the half length of the crack; \( dK_I \) and \( dK_{II} \) are the Mode I and Mode II SIFs at the crack tip caused by the concentrated force. For the plane stress problem, \( k = (3 - \nu)/(1 + \nu) \), where \( \nu \) is the Poisson’s ratio of the rock. Thus, the total SIFs caused by the non-uniform stress can be expressed as

\[
\begin{align}
K_I &= \sum_{i=1}^{n} \left[ \frac{P_i A_i}{2\sqrt{\pi a}} \left( \frac{a+x_i}{a-x_i} \right) + \frac{\tau_i A_i}{2\sqrt{\pi a}} \left( \frac{k - 1}{k + 1} \right) \right] \\
K_{II} &= \sum_{i=1}^{n} \left[ \frac{P_i A_i}{2\sqrt{\pi a}} \left( \frac{k - 1}{k + 1} \right) + \frac{\tau_i A_i}{2\sqrt{\pi a}} \left( \frac{a+x_i}{a-x_i} \right) \right]
\end{align}
\]

where \( K_I^e \) and \( K_{II}^e \) are the effective Mode I and Mode II SIFs, respectively; \( P_i \) and \( \tau_i \) are the normal and shear stresses acting on the element; \( i \) denotes the element ID number; \( A_i \) is the area of the

Fig. 1. Sketch of freezing-induced crack.

Fig. 2. Freezing pressure imposed on the crack. (a) Non-uniform stress. (b) Crack with concentrated force.

Fig. 3. Elements of the crack. (a) Sketch of a freezing-induced crack. (b) Elements of the crack interface.
element; and \( \chi \) is the distance from center of the element to the center of the crack.

### 2.2. Freezing pressure imposed on cracks

Crack initiation and propagation of rocks induced by freezing pressure are quite complex processes. If the crack is initially saturated, unfrozen water might migrate out of crack as the ice crystal grows and expands. On the other hand, if the crack is not saturated, the water could migrate to the freezing edge of the crystal. Theoretically, freezing pressure is significant only when the crack is confined or the freezing rate is high enough to form an ice plug that confines the unfrozen water, and consequently a significant amount of water can leak out of the crack tip (Fig. 4).

The volume expansion of confined water in crack tip can be written as

\[
\Delta V_{wi} = \eta \omega_c \left( V_{wi}^0 - Q_{iw} \right)
\]

where \( V_{wi}^0 \) is the initial water volume in the crack tip; \( \eta \) is the expansion coefficient of frozen water; \( \omega_c \) is the ratio of frozen particles in the confined water in the crack tip, a parameter to represent the percentage of water converted into ice; and \( Q_{iw} \) is the water volume migrating out of the crack, which can be obtained from the following formula:

\[
Q_{iw} = \int_0^t k(\nabla P - \phi_f \nabla T - \rho_w g) \cdot n \, dt
\]

where \( t \) is the time, \( k \) is the fluid permeability coefficient, \( \nabla P \) is the freezing pressure gradient, \( \phi_f \) denotes the segregation potential, \( \nabla T \) is the temperature gradient, \( \rho_w \) is the density of water, \( g \) is the gravity acceleration vector, and \( n \) is the unit direction vector of fluid water in the crack.

The maximum value of the freezing pressure in the crack tip can be written as follows:

\[
P_{i\text{max}} = \frac{\Delta V_{wi}}{V_{wi}^0} K_i
\]

where \( V_{wi}^0 \) is the initial water volume in the crack tip, and \( K_i \) is the bulk modulus of ice.

### 3. Stress distribution in the crack tip region

A crack, as shown in Fig. 5a, is considered, where \( \sigma_1 \) and \( \sigma_2 \) are the normal stresses imposed on the rock, and \( \beta \) is the angle between the crack plane and the direction of \( \sigma_3 \). The model is also considered as a plane stress problem, and the crack is initially water saturated. As the temperature decreases to the freezing point, a thin ice layer will appear in the crack, which would induce freezing pressure on the crack face. In addition, a shear stress and a normal stress would be induced by \( \sigma_1 \) and \( \sigma_2 \) on the crack plane, as shown in Fig. 5b.

The shear and normal stresses on the crack plane can be written as

\[
\begin{align*}
\tau_s &= \frac{1}{2\sqrt{2\pi r}} \left[ K_i (3 - \cos \theta) \cos \theta + K_{\text{fl}} (3 \cos \theta - 1) \sin \theta \right] \\
\sigma_{\theta\theta} &= \frac{1}{2\sqrt{2\pi r}} \left[ K_i \cos^2 \theta - \frac{3}{2} K_{\text{fl}} \sin \theta \right] \\
\tau_{r\theta} &= \frac{1}{2\sqrt{2\pi r}} \left[ K_i \sin \theta + K_{\text{fl}} (3 \cos \theta - 1) \right]
\end{align*}
\]

### 3.1. Composite crack under compression and shearing conditions

![Fig. 5. Composite crack under compression and shearing conditions.](image)

\[
\begin{align*}
\tau_s &= \frac{\sigma_1 - \sigma_2 \sin(2\beta) - \mu P}{2} H\left( \tau_0 - \tau_s^I \right) \\
P &= P_0 - \frac{\sigma_1 + \sigma_2 - \sigma_2 \cos(\pi + 2\beta)}{2} \left[ 1 + 2 \frac{\sigma_2}{P_0} \right] \\
\tau_0 &= \frac{\sigma_1 - \sigma_2 \sin(2\beta)}{2} - \mu P \\
H\left( \tau_0 - \tau_s^I \right) &= \begin{cases} 0 & (\tau_0 \leq \tau_s^I) \\
1 & (\tau_0 > \tau_s^I) \end{cases}
\end{align*}
\]

where \( \tau_s \) and \( P \) are the shear and normal stresses imposed on the crack plane, respectively; \( P_0 \) stands for the freezing pressure caused by volumetric expansion of water during phase transition; \( \mu \) and \( \tau_s^I \) represent the friction coefficient and shear strength of the crack propagation with transient sliding, respectively; \( H(\tau_0 - \tau_s^I) \) is a Heaviside step function; and \( \tau_0 \) is the shear stress component on the crack plane induced by \( \sigma_1 \) and \( \sigma_2 \).

When \( \tau_0 \leq \tau_s^I \), there is no relative sliding between the two faces of the crack, and the cohesion rises to offset \( \tau_0 \), leading to the reduction of the effective shear stress to zero. Under this condition, the model turns out to be a pure tension crack. On the other hand, when \( \tau_0 > \tau_s^I \), transient relative sliding will occur between the two faces of the crack with frictional resistance equal to \( \mu P \). Therefore, the result becomes that of a mixed mode of tensile and shear crack. Numerically, both the shear and normal stresses imposed on elements of the cracks would be obtained to calculate the SIFs. The stress field in polar coordinates near the crack tip is shown in Fig. 6.

According to theory of superposition for Mode I and Mode II cracks, the stress components in Fig. 6 can be expressed as follows (Yin, 1992):
4. Growth path of the wing crack

4.1. Direction of crack initiation

A wing crack grows out of the initial plane of the sliding crack when nucleated. The prediction of crack propagation behavior requires the computation of the SIF at each stage as well as the wing crack trajectory. The formulation and solution of this fracture mechanics problem can be very complicated (Matsuoka, 2008). This paper adopts the maximum circumferential stress criterion to analyze the freezing crack initiation criterion. It is assumed that crack initiation takes place in the direction vertical to the maximum circumferential stress. Therefore, the partial derivative of $\sigma_\theta$ to $\theta$ can be calculated as follows (Zhuang, 2006; Yang, 2010):

$$\sigma_{\theta\theta,\theta} = -\frac{3}{4\sqrt{2\pi a}}\frac{\sin \theta}{2} \left[ K_0 \sin \theta + 2K_{II} \left( \cos \theta - \sin^2 \frac{\theta}{2} \right) \right]$$  \(11\)

When $\sigma_{\theta\theta,\theta} \leq 0$ and $\sigma_{\theta\theta,\theta} < 0$, $\sigma_\theta$ reaches the maximum value. Since $0 \leq \theta < \pi$, we obtain $\cos(\theta/2) \neq 0$. Therefore, $K_0 \sin \theta + 2K_{II} \cos \frac{\theta}{2} = 0$. Then we have

$$K_0 \sin \theta + 2K_{II} \left( \cos \theta - \sin^2 \frac{\theta}{2} \right) = 0$$

$$K_0 \sin \theta + 2K_{II} \left( \cos \theta - 2 \sin^2 \frac{\theta}{2} \right) = 0$$  \(12\)

Dividing both sides of Eq. (12) by $2 \cos^2(\theta/2)$, we get the following equation:

$$K_0 \tan \frac{\theta}{2} + K_{II} \left( 1 - 2 \tan^2 \frac{\theta}{2} \right) = 0$$  \(13\)

or

$$2K_{II} \tan^2 \frac{\theta}{2} - K_0 \tan \frac{\theta}{2} - K_{II} = 0$$  \(14\)

The solution to Eqs. (13) and (14) can be written as

$$\theta_0 = \begin{cases} 2 \arctan \left( K_0 \pm \sqrt{K_0^2 + 8K_{II}^2} \right) & (K_{II} \neq 0) \\ 0 & (K_{II} = 0) \end{cases}$$  \(15\)

$$\sigma_{\theta\theta,\theta} = -\frac{3}{4\sqrt{2\pi a}} \left[ K_0 \left( \cos^2 \frac{\theta}{2} - 4 \sin^2 \frac{\theta}{2} \cos \theta \right) \right] - K_{II} \left( 5 \sin \theta \cos^2 \frac{\theta}{2} - 2 \sin^3 \frac{\theta}{2} \right)$$  \(16\)

According to the conclusion of Erdogan and Sih (1963), the crack initiation criterion can be expressed as follows:

$$\cos^2 \frac{\theta}{2} \left( \frac{K_1}{K_{IC}} \cos^2 \frac{\theta}{2} - \frac{3}{2} \frac{K_{II}}{K_{IC}} \sin \theta \cos \theta \right) = 1$$  \(17\)

or

$$K_1 \cos^2 \frac{\theta}{2} - \frac{3}{2} K_{II} \sin \theta \cos \theta = K_{IC}$$  \(18\)

where $K_{IC}$ is a critical parameter referred to as the crack toughness, reflecting rock’s ability to resist crack propagation. When the calculated $K_1$ exceeds $K_{IC}$, crack initiation commences. In the case of a pure tensile crack, $\theta_0 = 0$ and $K_1 = K_{IC} = \sigma(\pi a)^{1/2}$ when $K_{II} = 0$; whilst $\theta_0 = -70.53^\circ$, $K_{II} = 0.866K_{IC}$ when $K_1 = 0$.

4.2. Length of the wing crack

When the direction of crack initiation is determined, the crack length should be specified to demonstrate the path of crack growth. As shown in Fig. 7, the angle between the wing crack and main crack is designated as $\theta_0$. Crack propagation is regarded as unstable crack growth and each step is a transient process.

According to the theory proposed by Erdogan and Sih (1963), a wing crack can be considered as a pure tensile crack (Mode I). As shown in Fig. 7, $F_t$ is the traction force normal to the face of wing crack, which is caused by stresses imposed on the face of main crack at the intersection:

$$F_t = 2ab(\tau_s \sin \theta_0 + P \cos \theta_0)$$  \(19\)

where $b$ is the width of the crack. Considering the plane stress condition, we have $b = 1$.

The wing crack is considered to be a pure tensile crack and the SIF at the tip of the wing crack can be expressed as

$$K'_I = \frac{F_t}{\sqrt{\pi L}} = \frac{2a(\tau_s \sin \theta_0 + P \cos \theta_0)}{\sqrt{\pi L}}$$  \(20\)

The circumferential stress will gradually be reduced along the path of crack propagation; propagation ceases when the SIF is reduced to the critical value $K_{IC}$.
5. Coalescence mechanism of neighboring cracks

Neighboring cracks will affect each other under freezing pressures and far-field stresses. From a micromechanical point of view, the final stage of crack propagation is crack coalescence. The complex interaction of stress fields from numerous neighboring cracks plays an important role in the evolution of the crack network. The individual cracks may coalesce with each other as the wing cracks propagate in the rock. Three coalescence patterns are considered in the freezing cracked rock: (1) coalescence of tension cracks, which is considered as tensile coalescence, (2) coalescence of shear cracks, considered as shear coalescence, and (3) a mixed pattern of tensile and shear coalescence.

5.1. Tensile coalescence

As shown in Fig. 8, the tensile coalescence pattern is caused by tensile wing cracks. When the wing cracks coalesce with each other, or the tip of a wing crack intersects with another pre-existing crack, tensile coalescence occurs. The growth path of wing cracks can be derived from the analyses in Section 4.

5.2. Shear coalescence

As shown in Fig. 9, the rock bridge is broken due to shear stress. This case is considered as a plane stress problem. According to the equilibrium principle, the resultant forces in two directions equal zero as expressed by the following equation:

$$
\sum F_Y = \sum F_H
$$

As shown in Fig. 9, the following relations can be derived (Vásárhelyi and Bobet, 2000; Zhuang, 2006; Yang, 2010; Ning et al., 2011; Stakhovsky, 2011):

$$
\int_{2a_1} \left[ P_1(x_1) \sin \beta_1 + \tau_{\beta_1}(x_1) \cos \beta_1 \right] dx_1
+ \int_{2a_2} \left[ P_2(x_2) \sin \beta_2 + \tau_{\beta_2}(x_2) \cos \beta_2 \right] dx_2
+ (\sigma_{bl_1} - \tau_{bl_1}) = \sigma_2(2a_1 \sin \beta_1 + 2a_2 \sin \beta_2 + l_Y)
$$

(25a)

$$
\int_{2a_1} \left[ P_1(x_1) \cos \beta_1 - \tau_{\beta_1}(x_1) \sin \beta_1 \right] dx_1
+ \int_{2a_2} \left[ P_2(x_2) \cos \beta_2 - \tau_{\beta_2}(x_2) \sin \beta_2 \right] dx_2
+ (\sigma_{bl_1} - \tau_{bl_1}) = \sigma_1(2a_1 \cos \beta_1 + 2a_2 \cos \beta_2 + l_H)
$$

(25b)

Then, we can get

$$
\begin{align*}
\sigma_{bl_Y} + \tau_{bl_Y} &= A \\
\sigma_{bl_H} - \tau_{bl_H} &= B
\end{align*}
$$

(26)

where

$$
A = \sigma_2(2a_1 \sin \beta_1 + 2a_2 \sin \beta_2 + l_Y)
$$

(27a)

$$
B = \sigma_1(2a_1 \cos \beta_1 + 2a_2 \cos \beta_2 + l_H)
$$

(27b)

$$
\begin{align*}
\sigma_b &= \frac{(Bl_H + Al_Y)}{(l_H^2 + l_Y^2)} \\
\tau_b &= \frac{(Al_H - Bl_Y)}{(l_H^2 + l_Y^2)}
\end{align*}
$$

(28)

According to the Mohr–Coulomb criterion, when \( \sigma_b \) and \( \tau_b \) satisfy the following relation, shear coalescence occurs:
\[ \tau_b \geq c_b + \sigma_b \tan \phi \]  

(29)

where \(c_b\) and \(\phi\) are the cohesion and internal friction angle of the rock, respectively.

5.3. Mixed pattern of tensile and shear coalescence

As shown in Fig. 10, wing cracks will not intersect with each other. However, as the wing crack grows, the rock bridge between the cracks becomes thinner. The ability to resist shear damage declines.

Similar to the analysis of shear coalescence, we obtain the following equations:

\[
\int_{2a_1} \left[ P_1(x_1) \sin \beta_1 + \tau_{\beta_1}(x_1) \cos \beta_1 \right] dx_1 \\
+ \int_{2a_2} \left[ P_2(x_2) \sin \beta_2 + \tau_{\beta_2}(x_2) \cos \beta_2 \right] dx_2 \\
+ \frac{1}{\mu_b} (\sigma_b \sin \alpha - \tau_b \cos \alpha) = \sigma_2 (2a_1 \sin \beta_1 + 2a_2 \sin \beta_2 + h_f) \\
(30a)
\]

\[
\int_{2a_1} \left[ P_1(x_1) \cos \beta_1 - \tau_{\beta_1}(x_1) \sin \beta_1 \right] dx_1 \\
+ \int_{2a_2} \left[ P_2(x_2) \cos \beta_2 - \tau_{\beta_2}(x_2) \sin \beta_2 \right] dx_2 \\
+ \frac{1}{\mu_b} (\sigma_b \cos \alpha - \tau_b \sin \alpha) = \sigma_1 (2a_1 \cos \beta_1 + 2a_2 \cos \beta_2 + h_f) \\
(30b)
\]

Then, we get

\[
\sigma_b \sin \alpha + \tau_b \cos \alpha = A' \\
\sigma_b \cos \alpha - \tau_b \sin \alpha = B' \\
(31)
\]

\[
A' = \frac{\sigma_2}{\mu_b} (2a_1 \sin \beta_1 + 2a_2 \sin \beta_2 + h_f) \\
- \frac{1}{\mu_b} \int_{2a_1} \left[ P_1(x_1) \sin \beta_1 + \tau_{\beta_1}(x_1) \cos \beta_1 \right] dx_1 \\
- \frac{1}{\mu_b} \int_{2a_2} \left[ P_2(x_2) \sin \beta_2 + \tau_{\beta_2}(x_2) \cos \beta_2 \right] dx_2 \\
(32a)
\]

\[
B' = \frac{\sigma_1}{\mu_b} \left[ 2a_1 \cos \beta_1 + 2a_2 \cos \beta_2 + h_f \right] \\
- \frac{1}{\mu_b} \int_{2a_1} \left[ P_1(x_1) \cos \beta_1 - \tau_{\beta_1}(x_1) \sin \beta_1 \right] dx_1 \\
- \frac{1}{\mu_b} \int_{2a_2} \left[ P_2(x_2) \cos \beta_2 - \tau_{\beta_2}(x_2) \sin \beta_2 \right] dx_2 \\
(32b)
\]

The solution of Eq. (31) can be written as

\[
\sigma_b = A' \sin \alpha - B' \cos \alpha \\
\tau_b = B' \sin \alpha + A' \cos \alpha \\
(33)
\]

When \(\sigma_b\) and \(\tau_b\) satisfy Eq. (29), shear coalescence occurs.

6. A new algorithm for frost crack propagation

The ice pressure is the primary driving force of crack initiation and propagation. As shown in Fig. 11a, the ice-filled crack is defined as a double-faced structure with a certain open width, while the tip is simplified as an equilateral triangle. As shown in Fig. 11b, the elements intersected by the new crack can be classified into two categories: fully cracked elements (e.g. \(A_1\), \(A_2\), \(A_0\)) and partially cracked elements (e.g. \(A_3\)). The update of elements in the cracked region is the most important step for the evolution of the rock crack network. The algorithm for frost crack propagation should have the capability to update the grids in the cracked regions and to automatically distinguish the crack coalescence.

In order to achieve this, the following steps should be taken:

(1) The growing paths of each crack tip are calculated and defined according to Eqs. (15) and (23).
(2) Crack coalescence is judged by the topology method.
(3) The cracked elements are identified by the element ergodic loop method using a computing procedure.
(4) The cracked elements are updated and the parameter values of the new elements are defined for the next calculating cycle.

6.1. Basic concepts of the algorithm

In order to illustrate the algorithm explicitly, the following concepts should be introduced as shown in Fig. 12.

(1) Base points \(B_1-B_5\): the key points on the crack face that control the shape of the new crack.
(2) Edge points \(E_1-E_3\): the outer nodes of the new elements.
(3) Base lines \(l_{B_i}\) and \(l_{B_j}\): the connecting lines of the base points in sequence.

Fig. 10. Sketch of mixed pattern of tensile and shear coalescence.

Fig. 11. An example to illustrate elements. (a) The ice-filled crack, (b) Cracked elements.
(4) Edge lines $l_{E_1}$ and $l_{E_2}$: the connecting lines of the edge points in sequence.

(5) Mapping points $M_1$ and $M_2$: the auxiliary points generated on the base line according to the distribution of edge points. The mapping points would be taken as the inner nodes of the new elements.

(6) Auxiliary base line: the line defined near the crack tip to optimize the elements in the crack tip region.

(7) Scanning plane: the plane consists of base lines and auxiliary lines.

The growth path of the crack is predicted after a certain number of steps. The path contains information on the initiation angle and propagation length. As shown in Fig. 13, the tip point of the initial crack is defined as one of the base points of the new crack when $\theta < 30^\circ$ or $\theta \geq 30^\circ$.

6.2. Update of cracked elements

According to the distribution position, the edge points are divided into five zones, as shown in Fig. 14.

The methods used to update elements in each zone are different. The following conditions should be discussed:

(1) In zones I and V, $B_1(B_5)$ and the edge points are taken as the nodes of the new elements, as shown in Fig. 15a.

(2) In zones II and IV, $B_1(B_5)$ and $B_2(B_4)$ as well as mapping points are taken as the nodes of the new elements, as shown in Fig. 15b. The new elements will be generated in sequence by the computing procedure. The method of generating mapping points will be discussed in the following section.

(3) In zone III, the points $B_2, B_3, B_4$ and the edge points in zone III are taken as the nodes of the new elements, as shown in Fig. 15c.

(4) In the intermediate zone, the adjacent edge points of neighboring zones, as well as the closest base point, are taken as the nodes of the new elements, as shown in Fig. 15d.

In addition, the following two cases should be considered to avoid malformed elements:

(1) The edge point is too close to the base line, in which case the original element is not actually cracked. Therefore, the new element is quite difficult to generate, or malformed elements might be generated under this condition. In order to optimize the elements, the original element should be treated as a cracked one.

(2) The base point near the crack tip is too close to the edge line, as shown in Fig. 16a. Again, it is difficult to generate new elements. In this case the auxiliary line should be defined to enlarge the cracked zone near the tip.
6.3. Selecting cracked elements

When the original element is far away from the scanning plane, the element is not cracked, as shown in Fig. 17a. On the other hand, the element is considered to be cracked when the scanning plane crosses over the element plane, as shown in Fig. 17b. The coordinates of the nodes would be recorded and taken as edge points. Specifically, when the nodes lie on the scanning plane, or quite close to the scanning plane, they should be deleted, as shown in Fig. 17c and d.

Both the scanning plane and the original element plane consist of several straightline segments. Therefore, the relationship between the scanning plane and the original element plane can be determined by the line segments.

As shown in Fig. 18, the endpoint coordinates of the line segments are \((x_1, y_1), (x_2, y_2), (x_3, y_3)\) and \((x_4, y_4)\). The following equation set can be solved to get the intersection points (Shi, 1988):

\[
\begin{align*}
(x_1 + x_2 - x_1)R &= x_3 + (x_4 - x_3)R \\
y_1 + (y_2 - y_1)R &= y_3 + (y_4 - y_3)R
\end{align*}
\]  
(34)

where \(r\) and \(R\) are both constants.

Eq. (34) can be transformed into the following matrix form:

\[
\begin{bmatrix}
x_2 - x_1 & x_3 - x_4 \\
y_2 - y_1 & y_3 - y_4
\end{bmatrix}
\begin{bmatrix}
r \\
R
\end{bmatrix}
= \begin{bmatrix}
x_3 - x_1 \\
y_3 - y_1
\end{bmatrix}
\]  
(35)

If \((x_2 - x_1) x_3 - x_4 = 0\), the line segments are parallel to or overlapped with each other. Otherwise, we have

\[
r = \frac{x_3 - x_1}{y_3 - y_1} \quad R = r \frac{x_3 - x_1}{y_3 - y_1} = \begin{bmatrix}
x_3 - x_1 \\
y_3 - y_1
\end{bmatrix}
\]  
(36)

If \(0 \leq r \leq 1\) and \(0 \leq R \leq 1\), the line segments intersect with each other, as shown in Fig. 18a. Otherwise, there is no intersection, as shown in Fig. 18b.

6.4. Generating mapping points

Mapping points are the auxiliary points generated on the base line according to the distribution of edge points in zones II and IV. As shown in Fig. 19, the base vector of the base line is \(\vec{e}_g\), and the projection vector of \(B_1 E_i\) can be written as \(\lambda l \vec{e}_g\), where \(\lambda\) is a scalar and \(l\) is the length of the corresponding base line:

\[
\begin{align*}
\lambda l &= \overrightarrow{B_1 E_i} \\
&= \left(x_2^f - x_1^f, y_2^f - y_1^f\right)
\end{align*}
\]  
(37a)

The mapping point is defined by \(\overrightarrow{B_1 M_i} = \frac{1}{2}(\lambda l + \lambda_{i+1})\vec{e}_g\).

The direction angle of the vector in the global coordinate can be calculated as follows:

\[
\varphi_1 \left(\overrightarrow{B_1 E_i}\right) = \begin{cases} 
\arccos \left(\frac{x_2^f - x_1^f}{\overrightarrow{B_1 E_i}}\right) & (y_2^f - y_1^f > 0) \\
\arccos \left(\frac{x_2^f - x_1^f}{\overrightarrow{B_1 E_i}}\right) + \pi & (y_2^f - y_1^f \leq 0)
\end{cases}
\]  
(38)

6.5. Coalescence of cracks

As shown in Fig. 20, three cases of coalescence should be considered, i.e. coalescence at the trunks, tips and intersections, respectively. The predicted position should be adapted to the new one. The intersection points can be obtained according to Eqs. (35) and (36).

7. Case study

The most essential feature of frost damage is the evolution of a crack network, which is a complex process involving coupling of thermal, hydraulic and mechanical fields. As a result, proper simplifications are necessary. In this study, the following assumptions are made:
(1) Both the ice and the rock material satisfy the features of a Mohr–Coulomb material and their thermal conductivities are isotropic.

(2) Leakage of water during the freezing process can be neglected. The frozen ratio \( u(T) \), which represents the percentage of water that has been transformed into ice, satisfies the following relationship with temperature \( T \) (°C):

\[
\begin{cases}
0 & (T \geq 0 \degree C) \\
-T/20 & (-20 \degree C < T < 0 \degree C) \\
1 & (T \leq -20 \degree C)
\end{cases}
\] (39)

(3) The freezing point of water is assumed to be 0 °C and the supercooling stage of phase transition is neglected, suggesting that the freezing process starts as soon as the temperature drops to the freezing point. The volume expansion of water is 9% when the temperature drops from 0 °C to –20 °C. Using the method of equivalent thermal expansion coefficient to simulate the expansion of the ice, we obtain a value for the thermal expansion coefficient of ice of –0.0015 °C⁻¹. The negative sign stands for expansion rather than shrinking of the ice volume when the temperature drops (Kang et al., 2012b).

The effective SIFs of crack tips are calculated using a computing procedure. The direction and length of crack propagation are defined by Eqs. (15) and (23), respectively.

7.1. Modeling and calculating procedure

This paper uses ANSYS for modeling and meshing, and then the model is imported into FLAC³D by a conversion procedure. The modeling and calculating processes are shown in Fig. 21.

Using the above-mentioned method, the freezing model in FLAC³D is generated as shown in Fig. 22. The model contains three cracks, two of which are parallel to each other and perpendicular to the third one. The size of the block is 2.0 m × 2.0 m × 0.1 m. The length of the cracks is 0.5 m, and the open width is 2.0 mm. The cracks are initially water saturated. The main physico-mechanical and thermal parameters of the model are listed in Table 1.

Interface elements are generated in the gaps between the ice layer and crack faces. Parameters of the ice, such as cohesion and internal friction angle, are modified by a FISH procedure during calculation. Such parameters are roughly assumed to satisfy the following equation:

\[
F(T) = F_0 + u(T)(F_e - F_0)
\] (40)

where \( F_0 \) and \( F_e \) are the values of the parameters when the temperature \( T \) equals 0 °C and –20 °C, respectively.

7.2. Calculation results

A vertical stress of 1.0 MPa and a horizontal pressure of 0.5 MPa are applied to the model. According to the primary assumptions, the crack initiation angle and propagation length are defined by the theoretical analyses in Sections 2 and 3. \( K_{IC} \) is assumed to be \( 4.0 \times 10^5 \) Pa m¹/². A FISH procedure is modified to calculate the effective SIFs of each crack tip and to judge whether crack initiation starts. Furthermore, the angle and length of crack propagation are calculated. According to the new algorithm proposed in Section 6, the coalescence is demonstrated as shown in Fig. 23.

Another crack tip then propagates without coalescence; details of the element update are shown in Fig. 24. The separated new elements in the cracked region are shown in Fig. 25.

The final model after the first freezing process is shown in Fig. 26.
8. Discussion and conclusions

Frost damage is an important issue in rock weathering. The most essential feature of frost damage is crack propagation caused by temperature fluctuations around the freezing point of water. Considering the freezing-induced process in a rock mass, this paper analyzes the freezing crack propagation under compression-shear conditions. The freezing-induced cracking under compression-shear is considered to be a mixed mode of tensile cracks (Mode I) and shear cracks (Mode II). The expressions of Mode I and Mode II SIFs are given to predict the growth path of the freezing crack. Three coalescence patterns of adjacent frost cracks are considered, i.e. tensile coalescence, shear coalescence and a mixed pattern of tensile and shear coalescence.

A new algorithm for frost crack evolution is proposed, which could define the crack growth path and identify and update the cracked elements. The algorithm can cope with the evolution of multiple two-dimensional cracks. Further efforts should be made to generalize the method for three-dimensional cracks.

Finally, a numerical example is given to show the growth path of three freezing cracks under compression-shear conditions. The model is generated by ANSYS and is then imported into FLAC3D by a conversion procedure. The method utilizing the equivalent thermal expansion coefficient is applied to simulate the expansion of ice when frozen. A FISH procedure is modified to calculate the effective SIFs of each crack tip and to assess whether crack initiation starts, and both the angle and length of crack propagation are defined. Using this new algorithm, the cracked elements are continually updated. The work is completed automatically once the cycle step is specified. The model contains only three cracks and the result proves that the new crack-evolving algorithm is proficient in updating the cracked elements. However, when hundreds or thousands of cracks are planted in the model, the computing process could be more complicated, even though the problem could still be solved by the algorithm. Of course, the procedure would have to be modified and a larger matrix built to store details of each crack tip.

This research makes some simplifications and assumptions to study the crack propagation mechanism under freezing conditions. Real freezing cracks in rock masses can be much more complicated and, since it is hard to achieve a model that reflects the real scenario, a further study is needed.

Conflict of interest

We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

Acknowledgments

All authors gratefully acknowledge the financial support from the National Natural Science Foundation of China (Grant Nos.
41302237 and 41130742), and the State Key Development Program for Basic Research of China (Grant No. 2014CB046900).

References


