A modified maximum tangential tensile stress criterion for three-dimensional crack propagation

Dunfu Zhang1*, Weishen Zhu1, 2, Shucai Li1, 2, Bo Zhang1, Weidong Wang1

1 School of Civil Engineering, Shandong University, Jinan, 250061, China
2 Geotechnical and Structural Engineering Research Center, Shandong University, Jinan, 250061, China

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Abstract: The three-dimensional (3D) crack propagation is a hot issue in rock mechanics. To properly simulate 3D crack propagation, a modified maximum tangential tensile stress criterion is proposed. In this modified criterion, it is supposed that cracks propagate only at crack front in the principal normal plane. The tangential tensile stress at crack front in the principal normal plane in local coordinates is employed to determine crack propagation, which is calculated through coordinate transformation from global to local coordinates. New cracks will propagate when the maximum tangential tensile stress at crack front in the principal normal plane reaches the tensile strength of rock-like materials. Compared with the previous crack propagation criteria, the modified crack propagation criterion is helpful in calculating 3D crack stress intensity factor, and can overcome the limitations of propagation step determined by individual experiences in previous studies. Finally, the 3D crack propagation process is traced by element-free Galerkin method. The numerical results agree well with the experimental ones for a frozen resin sample with prefabricated 3D cracks.

Key words: principal normal plane; three-dimensional (3D) crack propagation; element-free Galerkin method; rock-like materials

1 Introduction

The simulation of crack propagation, especially the three-dimensional (3D) crack, is one of the most challenging issues in rock mechanics. Numerical methods, such as finite element method (FEM), boundary element method (BEM), etc., are widely used to analyze crack propagation (Gifford and Hilton, 1978; Gray et al., 1994). It is difficult, however, to automatically regenerate meshes during every crack propagation step. Element-free method (Belytschko et al., 1994) is a powerful alternative to simulate crack propagation, which does not require element structure but with enough nodes. An approximate function can be generated by using the least squares method. Boundary conditions can be implemented by using the Lagrange multiplier, but the numbers of unknowns will be increased (Krysl and Belytschko, 1995). Belytschko et al. (1995b) proposed a coupled FEM with element-free method to address the problems of crack propagation, but the process is somewhat complicated. Belytschko et al. (1995a) used element-free Galerkin method (EFGM) to study two-dimensional (2D) crack propagation, and this method has even been extended to 3D crack problems (Krysl and Belytschko, 1997; Deng et al., 2003; Huang et al., 2006).

For 3D crack propagation problem, the tensile stress criterion would be feasible given that a suitable fracture criterion should be selected first. The maximum principal stress criterion has its limitations in explaining some test results of crack initial propagation. As a result, the crack propagation cannot be properly traced. In this paper, the maximum tangential tensile stress at crack front in local coordinates is considered as the control parameter, with which the maximum principal stress criterion for 3D crack propagation problems can be modified.

2 The maximum tangential tensile stress criterion

Considering the properties of 3D crack rupture, Guo
(2007) made the following assumptions:

1. The rupture is generated along crack front.
2. The length of a new crack segment depends on the maximum tensile stress at crack front.
3. The new crack segments are almost hyperbolic, but not synchronous along crack front.
4. New crack segments are stable during each loading step under compression.

In the study of 3D crack rupture, it is very difficult to solve the dual variation in mathematics when using Griffith’s energy criterion. To overcome this difficulty, Palaiswamy and Knauss (1978) provided an equivalent tensile stress criterion, which can be described as follows:

1. Cracks propagate along the direction of the maximum tensile stress $\sigma_t$.
2. Cracks propagate when the maximum tensile stress $\sigma_t$ at crack front reaches the fracture strength $\sigma_b$ of the materials.

Further development of the tensile stress criterion involves the following two major points by Atkinson and Sammis (1987), Li (1990) and Li and Li (1992):

1. A rupture surface contains a set of crack lines. Each crack line propagates along the direction of the maximum tensile stress to form hyperbolic face.
2. The crack front is an isoline of the maximum principal stress.

Erdogan and Sih (1963) provided a maximum tangential tensile stress criterion with the following assumptions:

1. Cracks initially propagate along the direction of the maximum tangential tensile stress at crack front.
2. Cracks propagate when the tangential tensile stress exceeds a critical value.

With the above analyses, a new crack propagation criterion is proposed and defined as the modified maximum tangential tensile stress criterion, which can be depicted as follows:

1. It is assumed that cracks propagate only in the principal normal plane at a point along crack front in local coordinates.
2. The crack propagation direction can be determined along the direction of the maximum tangential tensile stress $\sigma_{\theta_{\text{max}}}$ in the principal normal plane.
3. When the maximum tangential tensile stress $\sigma_{\theta_{\text{max}}}$ at the front of line segment reaches the fracture strength $\sigma_b$ of materials, cracks start to propagate.
4. The line segments from crack front to the points with tangential tensile stress are defined as the crack propagation steps.

The local coordinates at crack front are shown in Fig. 1. The symbols $n$, $b$, $\tau$, $\theta$ and $r$ are defined as the principal normal, binormal, tangent, polar tangential and radial coordinates, respectively. The planes, $nt$, $tb$ and $bn$, are defined as the osculating plane, binormal plane and principal normal plane, respectively.

3 Tangential tensile stress at crack front

3.1 Description of 3D crack surface

The displacement of each calculation point on crack front in normal plane is denoted by line-segment. The small crack surface surrounded by four lines, i.e. former crack front, two adjacent line-segments and their connecting line at the tip, is defined as a crack rupture segment. Considering that a new crack rupture segment may bend and twist during 3D crack propagation, a series of small triangles are used to simulate the crack rupture segment, and its front is the connecting line of the two adjacent line-segments at the tip, as shown in Fig. 2. Plane sketches of 3D crack
propagation are illustrated in Fig. 3. The solid circle (●) denotes the tip of crack propagation step, which is completely free. The square box (□) denotes the vertex on the external border surface. The empty circle (○) denotes the convex or concave point on external border surface.

3.2 Node splitting

It is assumed that the distance between crack surfaces is very small for each propagation step. The nodes on the rupture path are checked. To describe the crack surface mathematically, the following methods are adopted:

1. If a node is located in the gap of the crack rupture segment, it is split into two nodes.
2. The split nodes are placed on two surfaces of crack rupture segment separately.
3. The connecting line of two split nodes is perpendicular to the crack rupture segment, as shown in Fig. 4.

3.3 Transform matrix

On the current crack front, a new local coordinate system is set up. As shown in Fig. 5, point \( I \) is located on former crack front. Points \( J \) and \( K \) are located on current crack front. Let \( \mathbf{r} \) be the tangent to crack front and \( \mathbf{e} \) be the assistant coordinate along the path from point \( I \) to \( J \). The principal normal \( \mathbf{n} \) and binormal \( \mathbf{b} \) can be calculated by \( \mathbf{n} = \mathbf{r} \times \mathbf{b} \) and \( \mathbf{b} = \mathbf{e} \times \mathbf{r} \), where \( \mathbf{r}, \mathbf{n} \) and \( \mathbf{b} \) are unit vectors.

The transform matrix \( A \) from a global coordinate system to a local coordinate system can be expressed as

\[
A = \begin{bmatrix}
  a_{11} & a_{12} & a_{13} \\
  a_{21} & a_{22} & a_{23} \\
  a_{31} & a_{32} & a_{33}
\end{bmatrix}
\]

The unit vector of tangent is

\[
\mathbf{r} = \frac{\begin{vmatrix}
  x_k - x_j \\
  y_k - y_j \\
  z_k - z_j
\end{vmatrix}}{\sqrt{(x_k - x_j)^2 + (y_k - y_j)^2 + (z_k - z_j)^2}}
\]

where \( i, j \) and \( k \) are the base vectors in global coordinate system.

According to the base vectors transform, the second row of \( A \) is

\[
\begin{bmatrix}
  a_{21} \\
  a_{22} \\
  a_{23}
\end{bmatrix}^\top = \frac{\begin{vmatrix}
  x_k - x_j \\
  y_k - y_j \\
  z_k - z_j
\end{vmatrix}}{\sqrt{(x_k - x_j)^2 + (y_k - y_j)^2 + (z_k - z_j)^2}}
\]

The unit vector of the assistant coordinate is...
According to Eqs. (2) and (4), the third row of \( \mathbf{A} \), which is the coordinate of binormal unit vector in global coordinate system, can be expressed as

\[
\begin{bmatrix}
{a_{11}} \\
{a_{12}} \\
{a_{13}}
\end{bmatrix} = \frac{1}{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}
\left[
\begin{array}{c}
(a_{23}(x_j - x_i) - a_{22}(z_j - z_i)) \\
(a_{21}(y_j - y_i) - a_{22}(x_j - x_i)) \\
(a_{21}(y_j - y_i) - a_{22}(x_j - x_i))
\end{array}
\right]^T
\]

(5)

The first row of \( \mathbf{A} \), which is the coordinate of principal normal unit vector in the global coordinate system, can be expressed as

\[
\begin{bmatrix}
{a_{21}} \\
{a_{22}} \\
{a_{23}}
\end{bmatrix} = \frac{1}{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}
\left[
\begin{array}{c}
(a_{21}(x_j - x_i) - a_{22}(z_j - z_i)) \\
(a_{21}(y_j - y_i) - a_{22}(x_j - x_i)) \\
(a_{21}(y_j - y_i) - a_{22}(x_j - x_i))
\end{array}
\right]^T
\]

(6)

### 3.4 Tangential tensile stress

In the normal plane of the local coordinate system, \( \mathbf{e}_n \) is the unit vector of the polar tangential coordinates:

\[
\mathbf{e}_n = [n, \tau, b] \begin{bmatrix} -\sin \theta \\ 0 \\ \cos \theta \end{bmatrix}
\]

(7)

According to the base vector transform, \( \mathbf{e}_n \) can be expressed as

\[
\mathbf{e}_n = [i, j, k] \begin{bmatrix} l \\ m \\ n \end{bmatrix} = [n, \tau, b] \mathbf{A} \begin{bmatrix} l \\ m \\ n \end{bmatrix}
\]

(8)

where \( l, m \) and \( n \) are direction cosines of \( \mathbf{e}_n \) in the global coordinate system.

Comparing Eqs. (7) and (8), we can obtain

\[
\begin{bmatrix} l \\ m \\ n \end{bmatrix} = \mathbf{A}^T \begin{bmatrix} -\sin \theta \\ 0 \\ \cos \theta \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} -\sin \theta \\ 0 \\ \cos \theta \end{bmatrix}
\]

(9)

The components of \( \tilde{\mathbf{\sigma}}_n \) along \( \mathbf{e}_n \) are expressed as

\[
\begin{bmatrix} \tilde{\sigma}_{nx} \\ \tilde{\sigma}_{ny} \\ \tilde{\sigma}_{nz} \end{bmatrix} = \left[ \begin{array}{ccc}
\sigma_x & \tau_{xy} & \tau_{xz} \\
\tau_{yx} & \sigma_y & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & \sigma_z
\end{array} \right] \begin{bmatrix} l \\ m \\ n \end{bmatrix} = \mathbf{A}_y \begin{bmatrix} l \\ m \\ n \end{bmatrix}
\]

(10)

According to Eqs. (7)–(10), \( \tilde{\mathbf{\sigma}}_n \) in the global coordinate system can be written as

\[
\tilde{\mathbf{\sigma}}_n = [n, \tau, b] \mathbf{A} \sigma_y \mathbf{A}^T \begin{bmatrix} -\sin \theta \\ 0 \\ \cos \theta \end{bmatrix}
\]

(11)

In the local coordinate system, \( \tilde{\mathbf{\sigma}}_n \) can be written as

\[
\tilde{\mathbf{\sigma}}_n = \sigma_y \mathbf{e}_n = [n, \tau, b] \sigma_n \begin{bmatrix} -\sin \theta \\ 0 \\ \cos \theta \end{bmatrix}
\]

(12)

Comparing Eqs. (11) and (12), we can obtain

\[
\sigma_n = \begin{bmatrix} -\sin \theta \\ 0 \\ \cos \theta \end{bmatrix} = \mathbf{A} \sigma_y \mathbf{A}^T \begin{bmatrix} -\sin \theta \\ 0 \\ \cos \theta \end{bmatrix}
\]

(13)

Using \( \{ -\sin \theta, 0, \cos \theta \} \) to left-multiply Eq. (13), the tangential tensile stress in the principal normal plane at crack front can be obtained:

\[
\sigma_n = \begin{bmatrix} -\sin \theta \\ 0 \\ \cos \theta \end{bmatrix} \mathbf{A} \sigma_y \mathbf{A}^T \begin{bmatrix} -\sin \theta \\ 0 \\ \cos \theta \end{bmatrix}
\]

(14)

### 4 Numerical simulations with element-free Galerkin method

#### 4.1 Mathematical description

The displacement matrix for a node in the local domain is

\[
\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} u_x' \\ v_x' \\ w_x' \\ u_y' \\ v_y' \\ w_y' \\ u_z' \\ v_z' \\ w_z' \end{bmatrix}^T
\]

(15)

Then, the fitted displacement for any point in the local domain is

\[
\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{N} \begin{bmatrix} u' \\ v' \\ w' \end{bmatrix}
\]

(16)

where \( \mathbf{N} \) is the matrix of shape function, and \( \phi_i \) is the shape function.

The strain at any point in the domain is
\[ \epsilon = \{ \epsilon_x, \epsilon_y, \epsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx} \}^T = Cu^* \]

\[ C = \begin{bmatrix} \varphi_\alpha & 0 & 0 \\ 0 & \varphi_\gamma & 0 \\ 0 & 0 & \varphi_\beta \\ \varphi_\alpha & \varphi_\gamma & 0 \\ 0 & \varphi_\beta & 0 \\ \varphi_\beta & 0 & \varphi_\beta \end{bmatrix} \]  \hspace{1cm} (i = 1, 2, \cdots, n) \]

where \( C \) is the derivative matrix of shape function.

The stress at any point in the domain is

\[ \sigma = D \epsilon = DCu^* \]  \hspace{1cm} (18)

where \( D \) is the matrix of elastic modulus.

The displacement boundary condition is imported with penalty parameter method. During the calculation with element-free method, the penalty parameter \( \beta \) is set to be \( 10^4 \)–\( 10^6 \) times of the elastic modulus \( E \). The corresponding simulation results are much more satisfactory.

The modified potential energy is expressed as

\[ \Pi = \iint_{\Omega} \frac{1}{2} \sigma \epsilon d \Omega - \iint_{\Gamma_f} u^T f d \Gamma_f - \iint_{\Gamma} \frac{\beta}{2} (s^T u - \overline{u})^T (s^T u - \overline{u}) d \Gamma_u \]  \hspace{1cm} (19)

where \( f \) is the body force; \( s \) is the cosine of the known displacement direction; \( \overline{u} \) and \( \overline{f} \) are the known displacement and surface force, respectively.

The discrete equation of the elastic system can be confirmed:

\[ Ku^* = F \]  \hspace{1cm} (20)

where

\[ K_{ij} = \begin{bmatrix} K_{ij} \end{bmatrix}_{(i) \times (j)} \ (i, j = 1, 2, 3) \]

\[ K_{ij} = \iint_{\Omega} C_i^T DC d \Omega + \beta \iint_{r_u} N_{\varphi}^T ss^T N_{\varphi} d \Gamma_u \]

\[ F_i = \iint_{\Omega} N_{\varphi}^T f d \Omega + \iint_{\Gamma_f} N_{\varphi}^T \overline{f} d \Gamma_f + \beta \iint_{r_u} N_{\varphi}^T s \overline{u} d \Gamma_u \]

4.2 Skills of nodes disposing

In the meshless method, the weight function and radius of influential area are the major factors influencing calculation precision. The node disposing has a great effect on the calculation precision as well. In this method, node distribution can be regular or irregular. However, the uniformity and density may influence the calculation precision directly (Jiang and Chen, 2005; Zhao, 2005).

4.3 Numerical simulations

4.3.1 A simply supported beam

Fig. 7 shows that a uniform load \( q = 10 \) kN/m is applied to a simply supported beam. The dimensions of the beam are 8 m in length and 1 m in height. The elastic modulus and the Poisson’s ratio are 30 MPa and 0.167, respectively.
The area of supported beam is divided into 55 uniformly distributed nodes, as shown in Fig. 8(a). The randomly distributed points are shown in Figs. 8(b) and (c). The fan-shaped zone is too large in Fig. 8(b), and too large or small in Fig. 8(c). There are 40 uniformly integral grids, and $4 \times 4$ Gauss points in each section. The force boundary is divided into 20 uniform segments, and in each segment there are 4 Gauss points. The numerical results of stress $\sigma$, for different node distribution types are listed in Table 1, in which accurate solutions are referred to Liu (1992).

<table>
<thead>
<tr>
<th>Node No.</th>
<th>Accurate solution</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-482</td>
<td>-478.513</td>
<td>-478.317</td>
<td>-480.901</td>
</tr>
<tr>
<td>2</td>
<td>-239.125</td>
<td>-237.744</td>
<td>-237.986</td>
<td>-239.030</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.008</td>
<td>-0.022</td>
<td>-0.005</td>
</tr>
<tr>
<td>4</td>
<td>239.125</td>
<td>237.975</td>
<td>237.996</td>
<td>239.028</td>
</tr>
<tr>
<td>5</td>
<td>482</td>
<td>478.243</td>
<td>478.228</td>
<td>480.907</td>
</tr>
</tbody>
</table>

Table 1 shows that the numerical results for case 1 agree well with the accurate solutions. The errors of cases 2 and 3 are greater than that of case 1. The results for case 1 are of priority. In case 1, the node densities of $9 \times 3$, $11 \times 5$ and $21 \times 9$ are selected with the same integral grids and integral rank. The numerical results in Table 2 show that the calculation precision cannot be much improved.

### 4.3.2 Cantilever beam bending

Fig. 9 shows a cantilever beam with rectangular section and its weight is not considered. Its dimensions are $8 \times 1 \times 1$ m ($L \times b \times h$). The elastic modulus and the Poisson’s ratio are $10$ GPa and 0.252, respectively. Its left boundary is fixed, its right boundary is free, and a load $q=10$ kN/m is applied along the thickness direction. The cantilever beam is divided into $33 \times 5 \times 5$ uniformly distributed nodes, and is composed of integral grids. Then, 48 nodes are included within each node zone to select a linear primary function, using 3-rank Gauss integral, and to determine the support radius of each node. The numerical results are shown in Fig. 10. They agree well with those of the classic beam theory (Liu, 1992).

#### 4.3.3 Stress concentration of a circle hole

Fig. 11 shows a circular hole located in the center of a finite rectangular board, and uniform load $q=10$ kPa is applied on two boundaries. Considering symmetry of the board, 1/4 of the board is considered. Its dimensions are 0.4 m in length, 0.3 m in height, and
0.05 m in thickness. The radius of the hole is 0.025 m. The elastic modulus and the Poisson’s ratio are 10 GPa and 0.252, respectively. The height or length of the board is at least 12 times radius of the hole, therefore the boundary conditions will not affect stress concentration along the hole boundary according to elastic theory. The calculation domain is divided into 360 nodes and $16\times12\times3$ integral meshes. 48 nodes are kept in a support domain of each node. 4-rank Gauss integral is carried out in each integral cell. A linear primary function is selected. The numerical results agree well with the results obtained by Xu (2003), as shown in Fig. 12.

5 3D crack propagation

Based on the modified maximum tangential tensile stress criterion and EFGM, crack propagation simulation can be implemented under uniaxial tensile stress for a horizontally central elliptical crack, a semicircular surface crack (side-rotating 45°), and a semicircular surface crack (pitching 45°) in a cube, respectively.

To improve the calculation precision near crack front, the expanded basis function $P^T(X) = \{1, x, y, z, \sqrt{r}\}$ is selected. The visual criterion is also adopted to embody the separation between nodes and Gauss integral points at both sides of crack.

The number of the nodes is constant within a support domain. The radius of a support domain may be determined for each node dynamically. This scheme can save computation time with the least squares method and increase computational efficiency of approximate function within the support domain, especially in the refined node zone.

5.1 Numerical simulations

5.1.1 Propagation of a horizontally central elliptical crack

As shown in Fig. 13, a horizontally elliptical crack is placed in the center of a cube. The dimensions of the cubic model are $0.8 \times 0.8 \times 0.6$ m (length x width x height). The long and short axes of the elliptical crack
are 0.4 and 0.2 m, respectively, and its thickness is 1 mm. The elastic modulus, the Poisson’s ratio and the tensile strength of the rock-like material are 5.6 GPa, 0.252 and 2.54 MPa, respectively. Uniformly distributed tensile stress \( q \) acts on the upper surface of the cube. The bottom surface is restrained on simple supported hinges.

The sample is loaded incrementally. As the load reaches a certain critical value, new cracks appear. When the increasing load stops, the crack propagation stops. The cracks will continue to propagate when the imposed load increases. The numerical results are shown in Fig. 14.

It can be seen from Fig. 14 that the propagations of horizontally central elliptical crack are self-similar. The elliptical rupture surface becomes coin-shaped finally. The numerical result agrees well with the result of BEM (Deng et al., 2003), as shown in Fig. 15.

5.1.2 Propagation of a semicircular surface crack (side-rotating 45°)

As shown in Fig. 16, a semicircular surface crack (side-rotating 45°) is placed in a cube. The radius of the semicircular crack is 20 mm. The elastic modulus, the Poisson’s ratio and the tensile strength of rock-like material are 5.6 GPa, 0.252 and 2.54 MPa, respectively. Uniformly distributed compressive stress acts on the upper surface of the cube. The bottom surface is restrained on simply supported hinges.

The applied load also increases step by step. The initial rupture surface is shown in Fig. 17. Numerical simulation shows that the initial rupture is observed within the domain of \([52.5°, 72.5°]\) measured from the diameter tip in the front face to crack tip in the original crack. The bending propagation is a major behavior with a slight micro-distortion. Numerical result agrees well with the test result obtained by Wong et al. (2002), using PMMA (polymethyl methacrylate) material, as shown in Fig. 18.
5.1.3 Propagation of a semicircular surface crack (pitching 45°)

As shown in Fig. 19, a semicircular surface crack (pitching 45°) is placed in a cube. The radius of the semicircular crack is 20 mm. The elastic modulus, the Poisson’s ratio and the tensile strength of rock-like material are 5.6 GPa, 0.252 and 2.54 MPa, respectively. Uniformly distributed compressive stress is imposed on the upper surface of the cube. The bottom surface is restrained on simply supported hinges.

The applied load increases step by step. The initial rupture surface and complete crack propagation surface are shown in Fig. 20.

Numerical simulation shows that 3D crack propagation can be categorized into the following three stages:

1. The initial stage. For convenience, the angle in the original crack surface is measured from x-axis to crack tip. In the place with the angle of 20°, crack rupture surface presents tensional shape, and the distortion is visible. Within the range of [0°, 20°], crack grows downwards and away from the original crack undersurface. The propagation angle keeps at ~32°. Here, the propagation presents a bending behavior. Within the range of [20°, 90°], crack grows upwards and close to the upper surface of original crack. The propagation angles decrease from 85.5° to 60°, which were measured in local coordinate system at crack front. The propagation presents a bending behavior with a mild distortion. The propagation surface stretches slowly to form a hyperboloid.

2. The middle stage. In the original crack, within the range of [0°, 25°], crack propagation stops, while within the range of [25°, 90°], crack grows continuously.

3. The final stage. In the original crack, in the place with the angle of 85°, crack rupture surface presents tensional shape. Within the range of [25°, 85°], crack rupture surface grows with bending and distortion behaviors to form an unclosed hyperboloid. Within the range of [85°, 90°], crack rupture surface curls towards the front face, and finally the tangent plane of the rupture surface is perpendicular to the load in far field. Thus, crack stops growing.

5.2 3D crack propagation test

An unsaturated resin (Guo, 2007) is used to prepare specimens with sizes of 70 mm×70 mm×140 mm. After frozen at about −10 °C, the material becomes brittle and possesses a linear stress-strain behavior. The material presents brittle characteristics at about −10 °C. Therefore, the frozen resin can be regarded as a rock-like material. To make the material more brittle,
all the specimens were frozen at −30 °C for 48 hours. At −30 °C, the elastic modulus, uniaxial compressive stress, rupture toughness and Poisson’s ratio are 5.6 GPa, 127 MPa, 0.55 MPa·m^{1/2} and 0.252, respectively. The ratio of compressive strength to tensile strength is about 5.0, i.e. the tensile strength is 25.4 MPa. A thin aluminum film with a thickness of 0.2 mm was used to create an internal prefabricated crack during casting and placed in the middle of the specimen by four cotton threads. The long and short axes of the prefabricated elliptic crack are 20 and 10 mm, respectively. The crack was embedded, pitching 45° to the applied compressive force. All the resin specimens were measured after being kept at −30 °C for 48 hours. The testing procedures followed the standard methods of American Society of Testing Materials (ASTM) and International Society for Rock Mechanics (ISRM). A linear variable differential transformer (LVDT) was installed on the specimen to measure the axial deformation. The load was controlled by the displacement of the pressing machine platen, and the displacement rate is 0.02 mm/s. The test results show that the crack initiates and propagates around the boundary of the prefabricated crack, and the maximum propagation length is approximately a half of the length of the prefabricated crack.

A typical propagation pattern of a 3D crack is shown in Fig. 21. The phenomenon of crack propagation agrees well with the above analytical results according to the modified maximum tangential tensile stress criterion.

6 Conclusions

In this study, a modified maximum tangential tensile stress criterion is proposed to properly simulate 3D crack propagation, and the numerical simulations and laboratory tests are conducted. Conclusions can be drawn as follows:

(1) Numerical results show that the assumptions for 3D crack propagation are reasonable. The proposed criterion can eliminate the errors and is helpful in calculating stress intensity factor.

(2) The 3D mathematical description of crack propagation has been well addressed. All the numerical simulations agree well with corresponding tests.

(3) The 3D elliptical crack propagations are self-similar. The elliptical crack front gradually extends to a circular shape. The crack surface will become coin-shaped, and eventually the specimen is broken into two pieces. The propagation process agrees well with those of tests and the result of BEM.

(4) The rupture surface of inclined semicircular surface crack is a hyperboloid. In local zone, the rupture surface presents tensional shape. When the tangent plane of the rupture surface is perpendicular to the load in far field, crack propagation stops.

References


