An extended displacement discontinuity method for analysis of stress wave propagation in viscoelastic rock mass

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Abstract: An extended displacement discontinuity method (EDDM) is proposed to analyze the stress wave propagation in jointed viscoelastic rock mass (VRM). The discontinuities in a rock mass are divided into two groups. The primary group with an average geometrical size larger than or in the same order of magnitude of wavelength of a concerned stress wave is defined as “macro-joints”, while the secondary group with a high density and relatively small geometrical size compared to the wavelength is known as “micro-defects”. The rock mass with micro-defects is modeled as an equivalent viscoelastic medium while the macro-joints in the rock mass are modeled explicitly as physical discontinuities. Viscoelastic properties of a micro-defected sedimentary rock are obtained by longitudinally impacting a cored long sedimentary rod with a pendulum. Wave propagation coefficient and dynamic viscoelastic modulus are measured. The EDDM is then successfully employed to analyze the wave propagation across macro-joint in VRM. The effect of the rock viscosity on the stress wave propagation is evaluated by comparing the results of VRM from the presented EDDM with those of an elastic rock mass (ERM) from the conventional displacement discontinuity method (CDDM). The CDDM is a special case of the EDDM under the condition that the rock viscosity is ignored. Comparison of the reflected and transmitted waves shows that the essential rock viscosity has a significant effect on stress wave attenuation. When a short propagation distance of a stress wave is considered, the results obtained from the CDDM approximate to the EDDM solutions, however, when the propagation distance is sufficiently long relative to the wavelength, the effect of rock viscosity on the stress wave propagation cannot be ignored.

Key words: stress wave propagation; extended displacement discontinuity method (EDDM); viscoelastic rock mass (VRM); micro-defect; macro-joint

1 Introduction

Rock mass is composed of rock blocks separated by discontinuities in different forms, geometries and sizes. The dynamic properties of a rock mass are always dominated by mechanical and geometrical properties of the discontinuities. The discontinuities (macro-joints and micro-defects) in a rock are shown in Fig.1(a). Investigation on the mechanical effect of discontinuities on stress wave propagation is important in solving problems of seismology, mining, dynamic stability of rock slopes and tunnels, etc..

Rock blocks do not behave perfectly elastic property during stress wave propagation due to the existence of various micro-defects, such as voids, micro-cracks and inclusions. Because of the stress wave interactions with the inherent micro-defects, defected rock material may dissipate energy of stress waves and behaves as a viscoelastic material. Numerous viscoelastic models have been used to analyze the mechanical properties of rock materials and for extrapolation beyond the range of experiment, such as logarithmic model [1], modified logarithmic model [2], power model [3], and exponent model [4, 5]. All these models used to consider the mechanical properties of rock blocks were obtained...
under a constant load. However, the viscoelastic parameters of rocks are frequency-dependent during stress wave propagation. Therefore, these models are insufficient to gain a deep understanding of dynamic viscoelastic responses of rock blocks. Impact and dynamic experiments should thus be considered, as indicated by Robertson [6]. Blanc [7] used the transient wave propagation method to determine the viscoelastic properties of solids, and proposed a general solution and a filter wave method. Bacon [8] presented an experimental method for considering the stress wave dispersion and attenuation in a viscoelastic bar, and obtained the propagation coefficients (attenuation coefficient and wave number) and phase velocity. The parameters are usually assumed independent of the incident wave frequency [9]. However, rocks and rock masses are always subjected to dynamic loads with a wide frequency range, from a few Hertz (e.g. earthquake) to thousands of Hertz (e.g. explosion). Frequency-independent models are insufficient to describe the dynamic responses of a rock mass. An efficient viscoelastic model with frequency-dependent parameters is totally necessary.

It has been realized that attenuation and dispersion phenomena occur when stress wave propagates across discontinuities including embedded joints, interfaces, cracks, pores, fractures and faults [10–14]. To investigate the effects of macro-joints on the wave propagation, the conventional displacement discontinuity method (CDDM) shown in Fig.1(b) [15–19] was commonly recommended. In such a method, each rock joint is considered as a non-welded interface with a large extent and small thickness related to the wavelength. It is assumed that the stresses across these joints are continuous, while the displacements are discontinuous. Although the deformation behaviors of joints can be considered as linear or nonlinear, the rock at the two sides of a joint is generally assumed to be intact and linearly elastic. Therefore, the effect of the inherent rock viscosity on wave propagation has been ignored. Quantitative representation of both rock viscoelastic property and its effect on stress wave attenuation has yet to be explored.

This paper proposes an extended displacement discontinuity method (EDDM) to analyze the effects of macro-joints on stress wave propagation through a viscoelastic rock mass (VRM), and the transmission and reflection coefficients based on the VRM model using the presented EDDM are analytically derived. A viscoelastic equivalent medium model is then introduced to investigate the effects of micro-defects on the wave propagation, as shown in Fig.1(c). The propagation coefficients (attenuation coefficient and wave number) and the dynamic viscoelastic moduli (storage modulus and loss modulus) of the viscoelastic equivalent medium are obtained by longitudinally impacting a cored long defected sedimentary rock bar using a pendulum. The transmitted waveforms for the same incident wave using the EDDM based on the VRM model and the CDDM based on the ERM model are compared. Discussions of the rock viscosity influence on the stress wave dispersion, transmission and reflection are also carried out.

2 The EDDM for VRM

Among many methods that study the effects of joints on stress wave propagation across a rock joint, the displacement discontinuity method (DDM) has been widely adopted. The EDDM is valid when the joints are planar, large in extent and small in thickness related to the wavelength. Thus, the stresses across a rock joint are assumed continuous, but the particle displacements across the joint are discontinuous.

Considering one-dimensional stress wave propagation, when the magnitude of stress wave is insufficient to mobilize the nonlinear deformation of the joint, the linear model can be used to describe the normal deformation behavior of the joint, i.e.

\[
\Delta u = \sigma / k_n
\]

(1)

where \( \Delta u \) is the closure of the joint, \( \sigma \) is the normal stress, and \( k_n \) is the normal stiffness of the rock joint.

Assuming the two opposite sides of the joint are viscoelastic, homogeneous and isotropic, the DDM can be expressed as boundary conditions in one-dimensional longitudinal wave propagation equations:

\[
\begin{align*}
\sigma_1 - \sigma_2 &= \Delta u \\
\sigma_1 &= \sigma_2
\end{align*}
\]

(2)

where \( u \) is the particle displacement with subscripts “ I” and “ II” referring to the two opposite sides of the joint shown in Fig.2.

The first term of Eq.(2) indicates that the difference of the displacements between the two sides of the rock
joint equals the closure (opening), and the second term of Eq.(2) requires that the stresses across the joint are continuous.

Using the trial solutions that satisfy the wave equations in terms of harmonic wave [20], the strain wave has the following form:

\[ \varepsilon = \varepsilon_0 \exp(i \omega t) \exp(-i k x - \alpha x) \]  

where \( \varepsilon \) denotes the one-dimensional harmonic particle strain in the \( x \)-direction with the amplitude of \( \varepsilon_0 \); \( t \) is the time; \( k \) is the wave number; \( \alpha \) denotes the attenuation coefficient; \( i \) denotes the imaginary sign; and \( \omega \) is the angular frequency, which can be obtained from frequency \( f \) by \( \omega = 2 \pi f \). The incident, reflected and transmitted strain waves are shown in Fig.2.

For the wave propagation in a viscoelastic medium, the dynamic constitutive relation can be obtained as

\[ \sigma(\omega) = E_{(I, II)}(\omega) \varepsilon(\omega) \]  

where \( E_{(I, II)} \) denotes the dynamic viscoelastic modulus, and it can be expressed as \( E'(\omega) = E(\omega) + iE''(\omega) \), where \( E'(\omega) \) is the storage modulus and \( E''(\omega) \) is the loss modulus.

Defining \( \gamma^2(\omega) = -\rho \omega^2 / E' \), where \( \gamma \) is the wave propagation coefficient, the solution for longitudinal stress wave propagation in a one-dimensional bar can be solved, and the transmitted and reflected waves for a normally incident P-wave are given by

\[ \varepsilon_{\text{tran}} = \frac{2}{2 + \gamma E' / k_n} \varepsilon_{\text{inc}} \]  
\[ \varepsilon_{\text{ref}} = \gamma E'/k_n \varepsilon_{\text{inc}} \]  

where subscripts “inc”, “ref” and “tran” denote the incident, reflected and transmitted waves, respectively, as shown in Fig.2.

It is noticed that, for a viscoelastic medium, the material parameter \( E' \) and the wave propagation coefficient \( \gamma \) are both frequency-dependent. And for the conditions when the viscosity of the rock material is ignored, the loss modulus becomes zero. In such cases, the transmitted and reflected waves can be degenerated to the solutions for the wave propagation in the ERM [21, 22]. Similar analytical process can be used to obtain the transmission and reflection coefficients for a shear wave.

3 Viscoelastic behaviors of micro-defected rocks

A series of impact tests were carried out to investigate the viscoelastic property of a micro-defected sedimentary rock. The experimental setup is shown in Fig.3. A sedimentary rock bar with a length of 129.80 cm and a diameter of 4.49 cm was used. The density of the sedimentary rock bar was measured to be 2 680.98 kg/m³. The integrity and homogeneity of the sedimentary rock bar were carefully examined. The end surface of the bar was grinded by a grinding machine before experiment to make sure a free stress condition at the free end. A pendulum steel hammer was used to generate a longitudinal pulse. The intensity of the pulse load can be adjusted by changing the swinging-angle of the hammer. A pair of diametrically opposite strain gauges located at the middle of the sedimentary rock bar shown in Fig.3 was used to measure the longitudinal strain. The history of the strain pulses in the rock bar was recorded at a time rate of \( 1.00 \times 10^7 \) s\(^{-1} \) (time resolution of \( 1.00 \times 10^{-7} \) s), which gave sufficient data points with sufficient accuracy to carry out the discrete Fourier transformation (DFT).
where \( \tilde{P}(\omega) \) and \( \tilde{N}(\omega) \) are the amplitudes of harmonic component waves traveling in the directions of increasing and decreasing \( x \), respectively, and they can be determined by initial and boundary conditions. Since the stress on the free end surface of the rock is zero, it can be obtained that \( \tilde{P}(\omega) = -\tilde{N}(\omega) \) on the free end of the rock bar [23]. The \( \gamma(\omega) \) is expressed as

\[
\gamma(\omega) = ik(\omega) + \alpha(\omega)
\]

where \( \alpha(\omega) \) is the attenuation coefficient, and \( k(\omega) \) denotes the wave number, as given in Eq.(3). The relations of \( \alpha(\omega) \) and \( k(\omega) \) versus \( \omega \) are shown in Fig.4.

![Fig.4 Wave propagation coefficients of sedimentary rock.](image)

The dynamic complex modulus of the rock is then calculated, and the storage modulus \( E'(\omega) \) and loss modulus \( E''(\omega) \) are determined by Bacon et al. [8, 9, 22–24], which can be written as follows:

\[
E'(\omega) = \rho \omega^2 (k^2 - \alpha^2) / (k^2 + \alpha^2)^2
\]

\[
E''(\omega) = 2 \rho \omega^2 k \alpha / (k^2 + \alpha^2)^2
\]

Figure 5 shows the relations of the storage modulus and the loss modulus against the component wave frequency. It can be obtained from Fig.5 that the mechanical properties of the sedimentary rock are highly frequency-dependent. The magnitude comparison of storage modulus and loss modulus shows that the viscosity of the sedimentary rock cannot be ignored during the impact analysis. In the high frequency range, the storage modulus and the loss modulus both become constant.

![Fig.5 Dynamic complex modululi of sedimentary rocks.](image)

4 Numerical examples

Using the DFT, any arbitrary incident waveform can be expressed as the sum of periodical harmonic component wave in the form of Eq.(7). The experimentally determined moduli are applied to calculating the stress wave propagation in the viscoelastic medium, and the EDDM will then be employed when a set of macro-joints is included in the viscoelastic medium.

In the following study, a half-cycle sinusoidal wave is assumed to be applied as the incident wave in the following form:

\[
E_{inc} = \begin{cases} I_0 \sin(\omega_k t) & (0 \leq t \leq \pi/\omega_k) \\ 0 & (t > \pi/\omega_k) \end{cases}
\]

where \( I_0 \) is the amplitude of the incident wave; \( \omega_k \) is the angular frequency of the incident wave and \( \omega_k = 2\pi f_k \), where \( f_k \) is the incident wave frequency. It is assumed that the defected rock material is with the wave propagation coefficients shown in Fig.4 and the dynamic viscoelastic moduli shown in Fig.5. The joints are assumed to be equally spaced and with a stiffness of \( k_n = 12.5 \text{ GPa/m} \).

4.1 Numerical results

The transmitted and reflected waves based on the presented VRM using the EDDM and those based on the conventional ERM using the CEEM [22] are compared in Figs.6 and 7 for different joint numbers and different joint spacings, respectively. To clearly evaluate the effects of micro-defects and macro-joints on the stress wave propagation, the transmitted wave across multi-joint can be divided into two components. The first one is the directly transmitted wave and the second one is the catching-up wave caused by multi-wave reflections among joints. And the transmission coefficient in the present study is defined as the ratio of directly transmitted wave amplitude to the incident wave amplitude.

In Fig.6, the joint spacing \( S = 0.05 \lambda_0 \), where \( \lambda_0 \) is the wavelength of the incident wave, while the joint number varies from one to four to investigate the effect of joint number and the distance of wave propagation on the transmitted waves. It can be observed from Fig.6 that the viscoelastic properties of defected rock material have distinct effects on the wave attenuation, and the difference of the transmitted and reflected waves based on ERM and VRM will increase as the joint number increases for the rock mass with a fixed joint spacing.
Fig. 6 Comparisons of transmitted and reflected waves by using ERM and VRM for different joint numbers ($S = 0.05 \lambda_b$).
Table 1 shows the transmission coefficients for the directly transmitted waves obtained based on the present VRM and the conventional ERM. The difference of the transmission coefficient is defined as

$$\Delta_T = \left| T_{\text{ERM}} - T_{\text{VRM}} \right| / T_{\text{VRM}}$$

<table>
<thead>
<tr>
<th>Joint number, $N$</th>
<th>$T_{\text{ERM}}$</th>
<th>$T_{\text{VRM}}$</th>
<th>$\Delta_T$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.95</td>
<td>0.91</td>
<td>4.40</td>
</tr>
<tr>
<td>2</td>
<td>0.90</td>
<td>0.82</td>
<td>9.76</td>
</tr>
<tr>
<td>3</td>
<td>0.85</td>
<td>0.75</td>
<td>13.33</td>
</tr>
<tr>
<td>4</td>
<td>0.81</td>
<td>0.68</td>
<td>19.12</td>
</tr>
</tbody>
</table>

It can be obtained from Table 1 that the wave propagation in the VRM attenuates faster than that based on the conventional ERM, and the difference increases as the joint number increases, which arises to 19.12% when stress wave propagates through a rock mass containing four joints with a joint spacing of $S = 0.05\lambda_0$.

Figure 7 shows the effect of joint spacing on the transmitted and reflected waves. Two joints are analyzed, and the joint spacings are $S = 0.005\lambda_0$, 0.02$\lambda_0$, 0.1$\lambda_0$, and 0.2$\lambda_0$, respectively. It can be obtained from Fig.7 that the difference of transmitted and reflected waves increases as the joint spacing increases for a rock mass with a fixed joint number.

Table 2 shows the comparisons of the transmission coefficients of directly transmitted waves obtained based on the VRM and ERM for six different joint spacings with a given joint number of $N = 2$. The difference of the transmission coefficients is the same as that defined in Eq.(12). From Table 2, the wave propagation distance has a significant effect on wave attenuation. When the joint spacing is small enough, the transmission coefficients obtained using VRM can
be replaced by those obtained by ERM for an approximate investigation, such as $\Delta t = 1.12\%$ when $S = 0.005\lambda_0$. However, the difference increases as the joint spacing increases. When the wave propagation through a rock mass with two joints and a joint spacing of $S = 0.2\lambda_0$, $\Delta t$ can be as large as 42.86%. In that case, the conventional ERM is no longer valid.

### 4.2 Discussions

#### 4.2.1 Effects of rock viscosity on the transmitted waveform

It is assumed that the stress wave in Eq.(11) is applied on the boundary of the rock mass with joint spacings of $S = 0.01\lambda_0$ and $0.15\lambda_0$, respectively. The transmitted wave through a rock mass containing one joint based on the present VRM and the conventional ERM, and the transmitted wave across an integrated rock with micro-defects are denoted as “VRM”, “ERM” and “VR”, respectively, as shown in Fig.8. “ERM” shows the effect of macro-joints on stress wave propagation and “VR” gives the effect of micro-defects. “VRM”, on the other hand, combines both effects.

From the comparison of the peak magnitudes of the transmitted waves, it can be observed that when the joint spacing is small, such as $S = 0.01\lambda_0$, the attenuation of transmitted wave is mainly caused by the macro-joints. However, when the joint spacing is sufficiently large, such as $S = 0.15\lambda_0$, the attenuation caused by the rock viscoelastic properties or micro-defects is larger than that caused by the macro-joints.

The effective propagation time for the stress wave is defined as the difference between the two time spots corresponding to the peaks of the incident and transmitted waves. Therefore, it can be obtained from Fig.8 that the existence of the joint will cause a distinct time delay when a stress wave passes through it. Both the present VRM and the conventional ERM can be used to consider the time delay caused by the rock joint.

#### 4.2.2 Effects of rock viscosity on the transmission coefficient

When an incident wave in the form of Eq.(11) propagates through a rock mass containing one to four joints, the relations between the transmission coefficient and the joint spacing based on the present VRM and the conventional ERM are shown in Fig.9. It is observed from Fig.9 that the transmission coefficient obtained by the ERM is independent of the joint spacing, while the transmission coefficient obtained by the VRM decreases as the joint spacing increases. The decreasing tendency of the transmission coefficient becomes small as the joint spacing increases. It can also be observed from Fig.9 that the difference between the transmission coefficients obtained by the ERM and the VRM increases as the joint spacing increases. The transmission coefficient obtained by the present VRM approaches to that obtained by the conventional ERM as the joint spacing approaches zero when the total joint number is fixed.

![Fig.8](image1)

**Fig.8** Effects of rock viscosity on transmitted waveforms.

![Fig.9](image2)

**Fig.9** Transmission coefficients for wave propagation through a rock mass with different joint spacings by using ERM and VRM.
The transmission coefficients for wave propagation through a rock mass with \( N = 1, 2, 3 \) and \( 4 \) for \( S = 0.005 \lambda_0, 0.05 \lambda_0, 0.1 \lambda_0, 0.15 \lambda_0, 0.2 \lambda_0 \) and \( 0.25 \lambda_0 \), are respectively shown in Fig.10. The transmission coefficients decrease as the joint number increases for both the VRM and the ERM. However, the wave attenuates fast when the viscoelastic properties are considered. It is also observed from Fig.10 that the conventional ERM is the special cases of the present VRM when the joint spacing approaches zero for a given joint number.

Fig.10 Transmission coefficients for wave propagation through a rock mass with different joint numbers by using ERM and VRM.

From Figs.9 and 10, it can be concluded that the practical wave attenuation phenomenon in a rock mass is caused by two effects: one is the existence of the macro-joints and the other is due to the viscoelastic properties caused by essential micro-defects. The conventional ERM can only reflect the effects of the macro-joints, therefore, the transmission coefficient keeps consistent for different joint spacings, as shown in Fig.9, and varies with respect to the joint number, as shown in Fig.10 [25]. However, the VRM can be used to consider the wave attenuation caused by both effects, which is able to represent more accurately the wave propagation distance effect, defined as joint spacing \( S \) multiplied by a joint number \( N \).

5 Conclusions

Stress wave propagation through jointed rock masses is experimentally and theoretically investigated. An EDDM for stress wave propagation through jointed VRM is proposed. The dynamic properties of the sedimentary rock are obtained from a series of impact tests. Frequency-dependent storage modulus, loss modulus and wave propagation coefficients are determined through the analysis of the experimental results. The transmitted and reflected waves are analytically derived by using the proposed EDDM. The effects of micro-defects and macro-joints on stress wave propagation attenuation are evaluated quantitatively. The following conclusions can be drawn based on the analytical results:

1. The impact tests show that the dynamic viscoelastic moduli (storage modulus and loss modulus) and the wave propagation coefficients (wave number and attenuation coefficient) of the sedimentary rocks are frequency-dependent. The influence of viscosity on the wave propagation through such rocks (e.g. sedimentary rock) cannot be simply ignored.

2. The analytical solutions of transmitted and reflected waves for a harmonic wave propagating across a rock joint in the VRM are deduced. The results show that the transmission and reflection coefficients based on the conventional ERM using the CDDM are the special cases of that based on the VRM using the present EDDM when the loss modulus of the rock material can be neglected.

3. The comparisons of the reflected and transmitted waves based on the ERM using the CDDM and the VRM using the EDDM show that the essential rock viscosity has significant effects on stress wave attenuation, while it has less effect on the effective propagation velocity.

4. The comparisons of transmission coefficients for a rock mass with different joint spacings and different joint numbers based on the ERM and the VRM show that the conventional ERM could only be used to consider the effects of macro-joints on the stress wave attenuation, while the VRM combines both effects of macro-joints and micro-defects.

5. When a stress wave propagates for a short distance, the ERM can replace the VRM to obtain an approximate solution. However, when the wave propagates for a sufficient distance related to the wavelength, the effect of rock viscosity on the wave attenuation cannot be ignored.

6. Although this study involves only limited cases for compressive waves, it demonstrates that the effects of essential rock viscosity on the wave propagation should be considered and the present viscoelastic model can be used to investigate the effects of micro-defects. The same analytical process can be used to analyze other kinds of waves, e.g. shear wave and surface wave propagating through a rock mass, and similar results can be expected.

References


