Phenomena and theoretical analysis for the failure of brittle rocks

Faquan Wu1, Jie Wu2*, Shengwen Qi1

1 Key Laboratory of Engineering Geomechanics, Institute of Geology and Geophysics, Chinese Academy of Sciences, Beijing, 100029, China
2 Engineering College, China University of Geosciences, Wuhan, 430074, China

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Abstract: Rockburst, an unstable failure of brittle rocks, has been greatly concerned in rock mechanics and rock engineering for more than 100 years. The current understanding on the mechanical mechanism of rockburst is based on the Coulomb theory, i.e. compressive-shear failure theory. This paper illustrates a series of tensile and tensile-shear fracture phenomena of rockburst, and proposes a methodology for the analysis of fracture mode and its energy dissipation process based on Griffith theory. It is believed that: (1) the fracture modes of rockburst should include compressive-shear, tensile-shear and pure tensile failures; (2) the rupture angle of rock mass decreases with the occurrence of tensile stress; (3) the proportion of kinetic energy in the released strain energy from a rockburst may be much larger than that transferred into surface energy; and (4) the understanding on the tensile and tensile-shear failure modes of rockburst may change the basic thinking of rockburst control, i.e. from keeping the reduction in initial compressive stress $\sigma_1$ to restricting the creation of secondary tensile stress.

Key words: failure of brittle rock; tensile-shear fracture; Griffith criterion; released strain energy; kinetic energy

1 Introduction

Rockburst, an unstable failure of brittle rocks, is frequently encountered in ground and underground engineering projects in Southwestern China recently, and it has brought forward a difficulty in engineering geology and rock mechanics. The factors concerned with rockbursts are commonly considered as follows:

(1) High crustal stress. It is well known that Southwestern China is feathered with complex geological structure and topography. Active tectonic movement induces high crustal stress and a series of zones are found with concentrated geological stress. Lots of measured in-situ stresses, $\sigma_1$, exceed 20 MPa, and the maximum value even reaches 57.27 MPa at the dam site of Xiaowan hydropower station that is located at Lancang River in Yunnan Province.

(2) Hard rock. Most of key structures are hosted in hard rock, such as granite, gneiss, marble or hard thick sandstone with a uniaxial compressive strength (UCS) over 60 MPa.

(3) Excavation. In the recent decades, many underground engineering projects are encountered with large-scale excavations. Taking Xiaowan hydropower station as an example, the height of cut slope at the dam site reaches 680 m, and the thickness of excavated rock exceeds 90 m; and for Jinping I hydropower station at Yalong River in Sichuan Province, the geometry of underground powerhouse is 28.9 m in width, 73 m in height, and 277 m in length.

Studies on rockburst have a long history since 7 accidents occurred in the gold mines of South Africa in 1908. After that, rockbursts happened frequently in deep mines in South Africa, Germany, Russia, Poland, England, Chile, Canada, and United States and other countries/regions. The earliest record of rockburst in China is a coal burst in Shengli coal mine in Fushun, Northeastern China in 1933. Since then, rockbursts in mines, diversion tunnels, highways, railways, underground powerhouses, etc., have been frequently reported. The most remarkable rockbursts are those recorded in Erlangshan highway tunnel, Qinling railway tunnel, and the diversion tunnel of Jinping II hydropower station.

Great progress has been made on theoretical
researches during the past decades. A series of criteria and evaluation methods for evaluating occurrence and intensity of rockbursts have been proposed, including Hoek method, Turchaninov method, Kidybinski method, Russense criterion, Gu method, Xu-Wang criterion, Hou criterion, and Tao criterion, etc. [1]. However, most criteria or methods are based on a common concept, i.e. ratio of stress to strength, indicating that rockburst is formed in a compressive rupture mode. As a necessary condition, researchers attempt to use the Coulomb theory to analyze the causes of the phenomena of rockbursts, to predict the extent of damage, and to seek possible engineering control measures. However, they have encountered serious theoretical difficulties. The difficulties in Coulomb theory when predicting rockburst mainly exist in the following aspects:

1. The Coulomb theory is mainly used to study compressive and shear failure. But in fact, rockburst in brittle rocks often takes place when the maximum principal stress $\sigma_1$ is far less than its UCS.

2. According to the Coulomb theory, rock rupture angle $\theta$, i.e. the angle between the fracture surface and $\sigma_1$, can be expressed as

$$\theta = 45^\circ - \frac{\varphi}{2}$$  \hspace{1cm} (1)

where $\varphi$ is the internal friction angle of rock, usually less than $60^\circ$. Therefore, the rupture angle $\theta$ should be theoretically larger than $15^\circ$. Unfortunately, it can often be as small as $3^\circ$–$5^\circ$ at excavation surface, which can be called “blade-like failure”. On the sidewall of underground space, this angle can even be $0^\circ$, namely, plate fracture.

Jaeger and Cook [2] tried to use the Griffith theory to explain the failure mechanism of brittle materials. However, the Griffith theory has not been appropriately used to explain the phenomenon of fracture and guide disaster control in a rock mass. The basic reason is that the Griffith theory has been normally thought to be only applicable to the mechanism of tensile or tensile-shear fracture. It is believed that most rock masses are under compressive and compressive-shear stress states, and large value of tensile stress will not be formed in rock mass because the discontinuities within rock mass do not have tensile strength. In fact, that is a misunderstanding.

This paper, based on Griffith theory, attempts to analyze the fracture phenomenon of brittle rock masses during excavation, including the small rupture angle, the mechanical mechanisms and the energy process, and tries to provide a theoretical basis for rock damage control.

2 Typical fracture phenomena of brittle rocks in excavation

In this section, we briefly introduce some rupture phenomena of rock masses induced by excavation during the construction of a large-scale hydropower station in Southwestern China.

2.1 Rupture phenomena in the dam foundation excavation of Xiaowan hydropower station

Xiaowan hydropower station is built on Lancang River in Yunnan Province. It is a double-curvature arch dam with a height of 292 m. The maximum height of the vertical excavation of foundation is 90 m. The dam foundation is hosted on gneiss with a UCS of 95–170 MPa, internal friction angle of $50^\circ$–$57^\circ$ and elastic modulus of 34–42 GPa. The principal stress $\sigma_1$ measured at slope varies from 20 to 35 MPa and the maximum value of $\sigma_1$ at a depth of 50 m below the bottom of valley reaches 57.37 MPa. Rock cores intensely have a rupture in the shape of thin disks.

A series of rockburst phenomena can be seen at the excavation surface. Besides, there are also onion-peeling and some other special failure modes. Figure 1 shows that the rupture angle is extremely small, even less than $5^\circ$.
Fig.1 Rockburst phenomena at the excavation surface of dam foundation of Xiaowan hydropower station.

2.2 Rupture of surrounding rock masses in a large-scale underground powerhouse

Jinping I hydropower station is built on Yalong River, Sichuan Province with a dam height of 305 m. Underground caverns include main powerhouse, transformer chamber and a series of tunnels. The size of the main powerhouse is 277 m × 73 m × 28.9 m. The UCS of the surrounding rock, thick layered marble, varies from 50 to 129 MPa. Elastic modulus is 20–45 GPa, and internal friction angle is 45°–56°. The maximum initial principal stress $\sigma_1$ around the plant is about 35.7 MPa. Ruptures during excavation of underground spaces mainly cover the following two types: (1) flake fracture by extrusion at the foot of top arch or the position with a bigger curvature; and (2) sheet cleavage at the sidewall (Fig.2).

(a) Flake fracture by extrusion at the foot of top arch or the position with a bigger curvature.

(b) Sheet cleavage at the sidewall.

Fig.2 Rupture of surrounding rock at the underground powerhouse of Jinping I hydropower station.

2.3 Common features of excavation-induced failure of brittle rock masses

Features of excavation-induced failure of brittle rock masses can be seen as follows:

(1) Shape of fragments. Most of the fragments present the shape of knife-like flakes, sheets and arch fractures by squeezing. The shear rupture angle can be as small as 3°–5°. The rupture angle for sheet cleavage at the sidewall is reduced even to 0°. These tiny fracture angles make the failure planes almost parallel to the excavated surface (Fig.3).

(a) Flake fracture by extrusion at the foot of top arch or the position with a bigger curvature.

(b) Sheet cleavage at the sidewall.

Fig.3 Orientation of fracture planes and its relations with the excavated surface.

(2) The way of movement. Apparent opening and shear displacement of the failure fragments reflect the features of tensile or tensile-shear movement. They can also verify the existence of secondary tensile stress state near the excavation surface.

(3) The last type of rock damage is usually accompanied by the fast release of the restored strain energy.

3 Mechanism of failure for brittle rocks induced by excavation

3.1 The occurrence of tensile stress state during excavation
It is commonly accepted that excavation may lead to the occurrence of tensile stress, and it has been verified by theoretical solutions and numerical simulations. Usually, tensile stress appears at some parts of excavated surfaces with small curvature, such as the sidewall of underground structures. For instance, calculation illustrates that a tensile stress of 1.127 MPa is induced at the straight wall in the main powerhouse of Jinping I hydropower station (Fig.4) [3]. In fact, Figs.1 and 2 have shown a series of rupture phenomena, indicating the existence of tensile stress. This possibly allows us to use the Griffith theory to analyze the brittle fracture phenomena caused by excavation.

3.2 Analysis of rock brittle fracture based on Griffith theory

(1) Failure stress conditions

As we know, the Griffith theory is applicable to the analysis of tensile and tensile-shear ruptures. The criterion can be expressed in the form of principal stress (taking compression as positive):

\[ \sigma = \sigma_1 \quad (3\sigma_3 + \sigma_1 \leq 0) \]  
\[ (\sigma_1 - \sigma_3)^2 = 8\sigma_1(\sigma_1 + \sigma_3) \quad (3\sigma_3 + \sigma_1 > 0) \]

where \( \sigma_1 \) is the tensile strength of rock. Equations (2) and (3) can be plotted in Fig.5.

We can see, from Fig.5, that the rock mass will not fail when the point \(( \sigma_1, \sigma_3 )\) is located at the right side of the strength curve in the coordinate system of \( \sigma_1 \) and \( \sigma_3 \). Once the stresses reach the critical state, or even cross the curve, the rock mass will be broken. So if the minimum principal stress in the rock mass is \( \sigma_3 = -\sigma_1 \), whatever the value of \( \sigma_1 \) is, the rock mass will be damaged in a tensile mode; while under the conditions of \( \sigma_3 \geq -\sigma_1 \) and \( 3\sigma_3 \leq \sigma_1 \leq 8\sigma_1 \), the point track of \(( \sigma_3, \sigma_1 )\) can fit the curve well, and rock mass will possibly suffer from tensile-shear failure, i.e.

\[ \sigma_1 \geq \sigma_3 + 4\sigma_1 + 4\sqrt{\sigma_1(\sigma_1 + \sigma_3)} \]  

Equation (4) infers that the rock mass can be broken under the condition of \( \sigma_1 < \sigma_3 \), and \( \sigma_1 \) doesn’t need to have a high value because the tensile strength of the rock mass is relatively lower. Taking the dam foundation of Xiaowan hydropower station as an example, the UCS of fresh gneiss is 168 MPa, internal friction angle \( \varphi = 56.6^\circ \), and its tensile strength is 8.85 MPa. In accordance with the brittle fracture theory referred above, the maximum principal stress \( \sigma_1 \), which leads to tensile-shear failure of rock, ranges from 26.55 to 70.8 MPa. We can imagine that, for a pure tensile failure, \( \sigma_1 \) can be much lower. According to the Coulomb theory, \( \sigma_1 \) can be written as

\[ \sigma_1 = \frac{1 + \sin \varphi}{1 - \sin \varphi} \sigma_3 + \sigma_c \]  

It is clear that \( \sigma_1 = 69.59 \) MPa at least is required for the fracture of the rock when considering the occurrence of the secondary tensile stress caused by excavation, i.e. \( \sigma_3 \geq -\sigma_1 \). However, such a large value of \( \sigma_1 \) is almost impossible to occur near the excavated surface of the dam foundation. That is why it is difficult to explain the failure of the rock mass with the Coulomb theory.

(2) Rupture angle

Equation (3) can be expressed in the form of \( \sigma - \tau \) as follows:

\[ \tau^2 = 4\sigma_1(\sigma_1 + \sigma_3) \]  

let

\[ T = \tau^2 - 4\sigma_1(\sigma_1 + \sigma_3) \]

and substitute \( \sigma \) and \( \tau \) with the functions of \( \sigma_1 \), \( \sigma_3 \), and \( \alpha = 90^\circ - \beta \), the expression of \( \alpha \) can be derived when \( T \) obtains its peak value. The shear rupture angle, \( \beta = \frac{\pi}{2} - \alpha \), can then be written as follows:

\[ \beta = \frac{1}{2} \arccos \frac{4\sigma_1}{\sigma_1 - \sigma_3} \]
The geometrical relationship between the rupture angle and shear strength curve is shown in Fig. 6. In Fig. 6, we can see that the rupture angle decreases when the normal stress on the failure plane decreases. And when the normal stress becomes tensile, the rupture angle is gradually reduced to 0°. This is the mechanical mechanism for the appearances of knife-like ruptures, sheet cracks and arch cracks.

Fig. 6 Diagram for rupture angle.

4 Energy analysis of fracture for brittle rocks by excavation

4.1 Failure stress condition in three dimensions

Murrell (1963) extended the Griffith theory to a three-dimensional situation [2] as

\[
\frac{1 + \nu}{6E} [\frac{2\nu}{E} + (\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2] = 24\sigma_r (\sigma_1 + \sigma_2 + \sigma_3)
\]

where \( \sigma_r \) is the normal stress on the failure plane and \( \nu \) and \( E \) are the Poisson’s ratio and the elastic modulus, respectively.

Taking the intersection curve between Eqs. (9) and (10) for example, an ellipse can be achieved in the plane of \( \sigma_3 = \sigma_2 - \sigma_1 \) as

\[
\frac{1 + \nu}{6E} [(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2] = 4(1 + \nu)\frac{\sigma_r}{E} (\sigma_1 + \sigma_2 + \sigma_3)
\]

Comparing Eq. (14) with Eqs. (12) and (13), we can get

\[
u \sigma_r = \frac{4\sqrt{6}(1 + \nu)}{\sqrt{(1 - 2\nu)}E} \sigma_3 \sqrt{u_c}
\]

where \( u_c \) is the energy for the shear fracture of a rock.

Meanwhile, as a reasonable extension, we can propose a tensile fracture energy criterion based on the pyramid tensile strength surface (Eq. (10)):

\[
u \sigma_r = \frac{4\sqrt{6}(1 + \nu)}{\sqrt{(1 - 2\nu)}E} \sigma_3 \sqrt{u_c}
\]

where \( u_c \) is the energy for the tensile fracture of a rock.

According to the elasticity theory, the tensile stress-induced strain energy can be written as

\[
u \sigma_r = \frac{1}{2} \sigma_i \varepsilon_i = \frac{1}{2} \sigma_i \frac{\varepsilon_i}{E} = \frac{\sigma_i^2}{2E}
\]

where \( \sigma_i \) and \( \varepsilon_i \) are the tensile stress and the related strain, respectively.

4.2 Three-dimensional strain energy criterion of failure

According to elastic mechanics, the distortion energy \( u_d \) and volumetric energy \( u_v \) of a unit can be written as

\[
u \sigma_r = \frac{1 + \nu}{6E} [(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2] = 24\sigma_r (\sigma_1 + \sigma_2 + \sigma_3)
\]

where \( \nu \) and \( E \) are the Poisson’s ratio and the elastic modulus, respectively.

On the other hand, multiplying both sides of Eq. (9) with \( (1 + \nu)/(6E) \), we can get a criterion in a form of distortion strain energy as follows:

\[
u \sigma_r = \frac{4\sqrt{6}(1 + \nu)}{\sqrt{(1 - 2\nu)}E} \sigma_3 \sqrt{u_c}
\]

Comparing Eq. (14) with Eqs. (12) and (13), we can get

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u \sigma_r = \frac{4\sqrt{6}(1 + \nu)}{\sqrt{(1 - 2\nu)}E} \sigma_3 \sqrt{u_c}
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\]

where \( \sigma_i \) and \( \varepsilon_i \) are the tensile stress and the related strain, respectively.

Similarly, the critical strain energy of tensile fracture, \( u_c \), can be written as \( u_c = \sigma_i^2/(2E) \). This infers that the Griffith criterion is not only a stress criterion, but also an energy criterion.

Similar to the strength criterion (Eqs. (9) and (10)), the energy criterion is supposed to be the combination of Eqs. (14) and (16). Namely, when any of the three principal stresses reaches the tensile fracture condition, the strain energy aroused by stresses will meet Eq. (16), and the failure of rock mass will take place preferentially. Otherwise, the distortion strain energy criterion in Eq. (14) or (15) will be used for the judgement of tensile-shear failure of rock masses.

Further analysis of above-mentioned energy criteria infers that while the tensile stress of rock mass, \( \sigma_3 \), increases in the vicinity of the excavated surface,
the energy $u_3 - u_u = \frac{1}{2E}(\sigma_j^3 - \sigma_i^3)$ in the criterion (Eq.(16)) will approach zero, thus the rock masses will tend to fail. Even Eq.(16) could not be satisfied, the distortion strain energy $u_i$ in Eq.(12) will increase and meanwhile the volumetric energy $u_v$ in Eq.(13) will decrease, which will make the rock mass much more easily be broken in tensile-shear mode. Anyway, the excavation will make the rock mass easier to fail.

4.3 Composition of released energy

The process of fracture for a rock mass is actually a course of energy release. When a surface-near rock is broken, the stress acting on it and the stored strain energy will both be released quickly. If the dissipated heat energy is neglected, the released strain energy will be transferred into surface energy $S$ and kinetic energy $K$. Then, for the energy of tensile fracture $u_t = u_{u_t}$ and shear fracture $u_s = u_{u_s}$, we have the following relationships:

$$u_{u_t} = S + K \quad \text{or} \quad \frac{\sigma_1^2}{2E} = S + K \quad (18)$$

$$u_{u_s} = S + K \quad \text{or} \quad 12(1+\nu)\frac{\sigma_1}{E}\sigma = S + K \quad (19)$$

The kinetic energy $K$ will decide the movement status of the ruptured rock unit, but the value of the parameter $K$ is decided by the energy needed for the creation of the new surface of fragments.

Figure 7 shows the geometry of a tensile failure unit. Let the micro unit be a cube with a side length $a = 1$. The excavation leads to a secondary tensile stress $\sigma_1 = -\sigma_1$, which is perpendicular to the excavated surface. The strain energy released by the rupture of the unit will be $\sigma_1^2 / (2E)$ according to Eq.(18). The fracture surface with an area $a^2 = 1$ will be parallel to the excavated surface as shown in Fig.7.

$$S = 2a\sigma_1^2 = 2a$$

the kinetic energy will be

$$K = u_{u_t} - 2\alpha = \frac{\sigma_1^2}{2E} - 2\alpha$$

(21)

For a specific rock block, $\sigma_1$ and $\alpha$ can be obtained by laboratory tests. In fact, $\alpha$ is the release rate of strain energy of rock, i.e. $G_{IC}$, which can be obtained from fracture mechanics:

$$\alpha = G_{IC} = \frac{1-v^2}{2E}K_{IC}^2$$

(22)

where $K_{IC}^2$ is the tested fracture factor of I-type crack.

Substituting Eq.(22) into Eq.(21), we can have

$$K = \frac{\sigma_1^2}{2E} = \frac{1}{2E}[\sigma_1^2 - 2(1-v^2)K_{IC}^2] \quad (23)$$

There are few tests at present to obtain both the parameters $\sigma_1$ and $K_{IC}$. The authors have tried some calculations listed in Table 1 for estimating the kinetic energy for different rocks based on test data or empirical data [4–9].

Table 1 Estimation on the kinetic energy released by fracture of rocks.

<table>
<thead>
<tr>
<th>Rock</th>
<th>$u_{u_t}$(N·m)</th>
<th>$2a = 2G_{IC}$(N·m)</th>
<th>$K$ (N·m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granite</td>
<td>353</td>
<td>65</td>
<td>288</td>
</tr>
<tr>
<td>Marble</td>
<td>430</td>
<td>42</td>
<td>388</td>
</tr>
<tr>
<td>Sandstone</td>
<td>242</td>
<td>78</td>
<td>164</td>
</tr>
</tbody>
</table>

The calculating results in Table 1 show that the released kinetic energy may be much larger than that of the surface energy. This is the reason that the failure of brittle rocks usually leads to a strong shock.

5 Engineering significance

Firstly, the methodology proposed in the paper can be used to estimate the location of different types of rockbursts. For instance, we can calculate the elastic stress field, as shown in Fig.8, and make further inference of the rockburst modes and their distribution at the excavated surface. The tensile stress $\sigma_1$ at the sidewall is around 1 MPa (Fig.8(a)), while the actual tensile strength is 5–9 MPa, thus there will be no pure tensile fracture of an intact rock unless there exist cracks previously. However, tensile-shear fracture is possible if the criterion (Eq.(3) or (9)) is satisfied. For the area of the right foot of top arch, $\sigma_1$ is compressive stress with a value larger than 5 MPa, definitely there is no possibility for tensile and tensile-shear fractures. However, when considering the distribution of $\sigma_1$ (Fig.8(b)), compressive fracture is possible at the position where $\sigma_1 > 8\sigma_1$, under two-
dimensional condition (see criterion, Eq.(3)) or \( \sigma_t > 12 \sigma \) for a three-dimensional condition (Eq.(9)).

Secondly, based on the calculated stress field of the surrounding rock mass, the failure criteria can be used to determine the range of rupture.

Finally, reducing the secondary tensile stress is obviously an effective way for the control of rockburst. Researchers usually try to avoid the fracture of the surrounding rock mass by keeping the initial stress state, especially the value of \( \sigma_3 \). However, the current measures such as anchor bar or cable could provide a very limited resistance. For instance (Fig.9), an array of cables with 2 000 kN and an interval of 3 m could only provide a prestress of 0.223 MPa, which is about 2.23% of the initial compressive stress of \( \sigma_3 \); while for the controlling of tensile failure, it can reach 22.3% of the secondary tensile stress at the sidewall.

6 Conclusions

From the above analyses, the following conclusions can be drawn:

(1) Failure phenomena and numerical simulation both have illustrated that excavation will induce the secondary tensile stress near the cutting surface, thus the tensile or tensile-shear fracture mechanism of rockburst is possible and reasonable.

(2) Analyses with both Griffith theory and Mohr stress circle have shown that the rupture angle of rock mass will decrease with the occurrence of tensile stress, and rupture angle tends to be zero while the tensile stress reaches the tensile strength of rock.

(3) Calculation based on theorem of energy conservation and the testing parameters, i.e. release rate of strain energy \( \alpha \), of brittle rock with the fracture mechanics method infers that the proportion of kinetic energy in the released strain energy from a rockburst may be much larger than that of surface energy for the fracture of brittle rocks.

(4) The understanding of the tensile and tensile-shear failure modes of rockburst may lead to a change in the basic thinking in rockburst control, i.e. from keeping the reduction of initial compressive stress \( \sigma_3 \) to restricting the creation of secondary tensile stress, which may be a much more effective way.

References


