Sensitivity and inverse analysis methods for parameter intervals

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Abstract: This paper proposes a sensitivity analysis method for engineering parameters using interval analyses. This method substantially extends the application of interval analysis method. In this scheme, parameter intervals and decision-making target intervals are determined using the interval analysis method. As an example, an inverse analysis method for uncertainty is presented. The intervals of unknown parameters can be obtained by sampling measured data. Even for limited measured data, robust results can also be obtained with the inverse analysis method, which can be intuitively evaluated by the uncertainty expressed in terms of an interval. For complex nonlinear problems, an iteratively optimized inverse analysis model is proposed. In a given set of loose parameter intervals, all the unknown parameter intervals that satisfy the measured information can be obtained by an iteratively optimized inverse analysis model. The influences of measured precisions and the number of parameters on the results of the inverse analysis are evaluated. Finally, the uniqueness of the interval inverse analysis method is discussed.

Key words: interval analysis method; sensitivity analysis; reversible inverse analysis method; iteratively optimized inverse analysis method

1 Introduction

The purpose of a sensitivity analysis is to evaluate the extent of the changes that occur in a decision-making target or output by varying one or more of the uncertain input factors. Based on such a sensitivity analysis, the enduring capability or stability of the target can be assessed and the predictions can be made regarding changes that might occur in the output when large changes happen to the input. Traditionally, sensitivity analysis methods have been proposed based on stochastic theory and fuzzy theory. However, more recently developed interval analysis methods are gaining their acceptance. However, without sufficient experimental data to generate suitable probability density functions, these traditional methods are not likely to produce sufficiently accurate results. Elishakoff [1] discussed such uncertainties.

The interval analysis method, introduced by Moore and Yang [2], is based on non-probabilistic interval sets. This method assumes that the uncertainties of all loads and resistance parameters are bounded from above and below. By applying the interval analysis method, a sharp interval set that includes all feasible solutions can be obtained.

By far, the most widely investigated sensitivity analysis techniques include the Bayesian inverse analysis method, the maximum likelihood inverse analysis method, the Kalman filter inverse analysis method and the fuzzy inverse analysis method. For these methods, the deterministic stiffness matrix of the system is assumed to be known and the distribution of external loads is identified by interval displacements using the Lagrange multiplier method by Nakagiri and Suzuki [3]. These techniques all address the concept of interval inverse analysis, which constrains a model using the variable metric method. Such a method was used by Wang et al. [4, 5] to analyze the initial stress field, elastic constants and vibrational parameters of tunnel walls in a concrete dam. However, the model is sensitive to the initial values of the parameters, which in turn results in different initial values being used for the computation. Based on the perturbation method,
another interval inverse analysis model was proposed by Liu et al. [6] to analyze the elastic constants of the rocks in tunnel walls. The central values of the interval parameters are first determined by a conventional inverse analysis method, then the uncertainties of the interval parameters are constrained by the perturbation formulæ.

In this paper, the interval analysis method is briefly outlined. The formulæ needed to compute the sensitivity factors of the parameters are derived based on interval theory. A method for obtaining the parameter intervals and decision-making targets is presented. An iterative optimization model based on inverse analysis is then proposed to identify the parameter intervals. Finally, conditions needed to determine the existence or convergence of a solution are summarized.

2 Brief introduction to the interval analysis method

An interval number is defined by bounded sets of real numbers, which can be expressed as \( X^i = [x_i, \bar{x}_i] = \{ x | x \leq x \leq \bar{x}_i \} \), where \( x \) and \( \bar{x} \) are its two boundaries. The uncertainty \( \Delta X = (\bar{x} - x) / 2 \), the central value \( X^c = (x + \bar{x}) / 2 \), and the interval number \( X^i = [X^c - \Delta X, X^c + \Delta X] \) are defined. The variation coefficient of \( X^i \) is defined by \( \delta = \Delta X / X^c \), and its absolute value is defined as

\[
| X^i | = \max \{|x_i|, |ar{x}_i|\} 
\]

(1)

For the interval vector \( V^i = \{X^i_1, X^i_2, \cdots, X^i_n\}^T \), its norm is

\[
|| V^i || = \max \{|X^i_1|, |X^i_2|, \cdots, |X^i_n|\} 
\]

(2)

The central value and uncertainty of \( V^i \) are

\[
V^c = \{X^c_1, X^c_2, \cdots, X^c_n\}^T 
\]

(3)

\[
\Delta V = \{\Delta X_1, \Delta X_2, \cdots, \Delta X_n\}^T 
\]

(4)

It is assumed that

\[
\Delta V^i = \{\Delta X^i_1, \Delta X^i_2, \cdots, \Delta X^i_n\}^T 
\]

(5)

where \( \Delta X^i_i = [-\Delta X_i, \Delta X_i] (i = 1, 2, \cdots, n) \). Thus,

\[
V^i = V^c + \Delta V^i. 
\]

Similar expressions exist for an \( n \times n \) interval matrix \( A^i \), and its central value and uncertainty are

\[
A^c = (a^c_{ij}) \quad \Delta A = (\Delta a^c_{ij}) 
\]

(6)

Therefore, we have \( A^i = A^c + \Delta A^i \), where \( \Delta A^i = \{\Delta a^c_{ij}\} \), \( \Delta a^c_{ij} = [-\Delta a_{ij}, \Delta a_{ij}] \).

For the interval numbers \( X^i = [x_i, \bar{x}_i] \) and

\[
Y^i = [y_i, \bar{y}_i], 
\]

the basic operations are

\[
X^i + Y^i = [x_i + y_i, \bar{x}_i + \bar{y}_i] 
\]

(7)

\[
X^i - Y^i = [x_i - \bar{y}_i, \bar{x}_i - y_i] 
\]

\[
X^i \cdot Y^i = [\min\{|xy_i, x\bar{y}_i, \bar{x}y_i, \bar{x}\bar{y}_i\}, \max\{|xy_i, x\bar{y}_i, \bar{x}y_i, \bar{x}\bar{y}_i\}] 
\]

\[
X^i / Y^i = [x_i / y_i, \bar{x}_i / \bar{y}_i] = [\frac{1}{\bar{y}_i}, \frac{1}{y_i}] (0 \notin [y_i, \bar{y}_i]) 
\]

According to the basic operations presented above, we can find that the commutative and associative laws for addition and multiplication hold true. However, other operation laws (such as the distributive and counter-balance laws) present the inclusive forms, e.g.

\[
X^i (Y^i \pm Z^i) \subseteq X^i Y^i \pm X^i Z^i 
\]

(8)

Detailed interval computations are shown in Refs.[7, 8]. The interval operations may result in interval extension [9]. We can guarantee a sharp interval by the perturbation method [10]. Some other methods have been brought forward to deal with interval extensions, such as combination monotone method [11] and truncation method [11, 12].

The interval finite element method (FEM) is based on a combination of the interval analysis method and a traditional FEM. The response intervals of a linear-elastic beam were solved using triangle inequality and linear programming [13]. The interval-truncation approach was proposed to limit the interval extension for realistic and accurate solutions [11]. Based on perturbation theory, the interval matrix perturbation method and the interval parameter perturbation method were proposed to estimate the static displacement bounds of structures with interval parameters [10, 14, 15]. Considering the interactions among uncertain parameters, an improved interval perturbation method was proposed [16]. Based on some of the properties and operation laws of the interval number, a static governing equation for an n-freedom uncertain displacement field was transformed into (2n)-order linear equations [17]. Starting from the elements, the interval parameter perturbation method was proposed for the response intervals of a linear-elastic beam [18], and the sub-interval perturbed finite element method was proposed and applied to anti-slide stability analysis.
More recently, the dynamic responses of uncertain structures with interval parameters have been widely investigated [20, 21].

3 Parameter sensitivity interval analysis method

3.1 Sensitivity factor matrix

For a structure with \( n \) model parameters [22], the parameter vector is defined as \( V = \{x_1, x_2, \ldots, x_n\} \), where \( x_j = [x_j^L, x_j^U] \) (\( j = 1, 2, \ldots, n \)). In addition, the decision-making target vector \( Y = \{y_1, y_2, \ldots, y_m\} \) is composed of \( m \) decision-making targets, where \( y_i = [y_i^L, y_i^U] \) (\( i = 1, 2, \ldots, m \)). Because one model parameter vector \( V \) must correspond to one decision-making target vector \( Y \), the mapping of \( V \rightarrow Y \) is established and the relationships between \( x_j \) and \( y_i \) are given by

\[
\begin{align*}
y_{i1} &\leq \theta_{11} x_1 + \theta_{12} x_2 + \cdots + \theta_{1n} x_n \\
y_{i2} &\leq \theta_{21} x_1 + \theta_{22} x_2 + \cdots + \theta_{2n} x_n \\
&\vdots \\
y_{im} &\leq \theta_{m1} x_1 + \theta_{m2} x_2 + \cdots + \theta_{mn} x_n 
\end{align*}
\]

(9)

where \( \theta_{ij} (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n) \) is the integrative influence factor of the \( j \)-th model parameter on the \( i \)-th decision-making target. If \( V \rightarrow Y \) is nonlinear, then

\[
\begin{align*}
y_{i1} &= y_{i1}(x_1, x_2, \ldots, x_n) \\
y_{i2} &= y_{i2}(x_1, x_2, \ldots, x_n) \\
&\vdots \\
y_{im} &= y_{im}(x_1, x_2, \ldots, x_n) 
\end{align*}
\]

(10)

Applying the first-order Taylor series, we obtain

\[
\begin{align*}
y_{i1} &= y_{i1}(x) + \frac{\partial y_{i1}}{\partial x_1}(x_1 - x_1^i) + \frac{\partial y_{i1}}{\partial x_2}(x_2 - x_2^i) + \cdots \\
&\frac{\partial y_{i1}}{\partial x_1}(x_1 - x_1^i) \\
y_{i2} &= y_{i2}(x) + \frac{\partial y_{i2}}{\partial x_1}(x_1 - x_1^i) + \frac{\partial y_{i2}}{\partial x_2}(x_2 - x_2^i) + \cdots \\
&\frac{\partial y_{i2}}{\partial x_2}(x_2 - x_2^i) \\
&\vdots \\
y_{im} &= y_{im}(x) + \frac{\partial y_{im}}{\partial x_1}(x_1 - x_1^i) + \frac{\partial y_{im}}{\partial x_2}(x_2 - x_2^i) + \cdots \\
&\frac{\partial y_{im}}{\partial x_m}(x_m - x_m^i) 
\end{align*}
\]

(11)

where \( \theta_{ij} = \frac{\partial y_{ij}}{\partial x_j} \), \( V = \{x_1^i, x_2^i, \ldots, x_n^i\} \). Because of the difference in the dimensions of the model parameter, it is very difficult to determine \( \theta_{ij} \). Thus the influence of a single model parameter on a certain decision-making target is of concern.

Given the boundary influence value interval of the model parameter \( x_j \in [x_j^L, x_j^U] \) (\( j = 1, 2, \ldots, n \)) and the decision-making target \( y_i = [y_i^L, y_i^U] \) (\( i = 1, 2, \ldots, m \)), where \( [y_{ij}^L, y_{ij}^U] \) is one subset of \([y_i^L, y_i^U] \), the following parameter is defined:

\[
\theta_{ij}' = \frac{y_{ij}^U - y_{ij}^L}{y_i^U - y_i^L}
\]

(13)

where \( \theta_{ij}' \) is the independent influence factor of the model parameter \( x_j \) on the decision-making target \( y_i \). Because \( [y_{ij}^L, y_{ij}^U] \) is one subset of \([y_i^L, y_i^U] \), then \( 0 < \theta_{ij}' \leq 1 \). When \( \theta_{ij}' \) is close to 0, this indicates that the model parameter \( x_j \) has almost no influence on the decision-making target. On the contrary, when \( \theta_{ij}' \) is close to 1, the model parameter \( x_j \) has a very significant influence on the decision-making target \( y_i \).

Therefore, the sensitivity factor matrix \( \theta' \) of model parameter is described by

\[
\theta' = \begin{bmatrix}
\theta_{11}' & \theta_{12}' & \cdots & \theta_{1n}' \\
\theta_{21}' & \theta_{22}' & \cdots & \theta_{2n}' \\
\vdots & \vdots & \ddots & \vdots \\
\theta_{m1}' & \theta_{m2}' & \cdots & \theta_{mn}'
\end{bmatrix}
\]

(14)

The influence of each model parameter on each decision-making target is different. Furthermore, the influence of the same model parameter on different decision-making targets can vary considerably. From the sensitivity factor matrix \( \theta' \), the column vectors show the independent influence factors of each model parameter.
parameter on all the decision-making targets, the value of each factor reflects the influence of the model parameter on the corresponding decision-making target. The row vectors show the influence of all the model parameters on the same decision-making target: the larger the values of $\theta_i^y$ are, the greater the influence of the model parameters is. The sensitivity analysis method, to some extent, can consider the inter-relationship of the model parameters.

3.2 Determination of the boundary influence value interval and decision-making target interval

The boundary influence value interval $[y_{i,j}, \bar{y}_{i,j}]$ of the model parameter $x_j$ on decision-making target $y_i$ can be investigated by interval finite element optimization analysis when $x_j$ is an interval number but the other parameters are assumed to be constant and equal to the interval medium-values.

The decision-making target interval $[y, \bar{y}]$ can be determined using a similar method when all the parameters adopt interval numbers.

3.3 Discussion on parameter intervals

During the sensitivity analysis, the difference of the parameters’ dimensions may induce a variance in the sensitivity degree. Therefore, the parameters’ dimensions must be transformed to some extent, as illustrated by the following example.

Given $z = x^2 + 10y$, where $x \in X^1$, $y \in Y^1$, $z = Z^1$, $X^c = Y^c = 10$. $z$ is a binomial function of $x$, but a simple function of $y$, so the dimensions of $x$ and $y$ are different. If $\delta_x = 0.1$, we can get $X^1 = [9, 11]$, $Y^1 = [9, 11]$. By interval operation, the decision-making target interval $Z^1 = [171, 231]$ is obtained, as are the boundary influence value intervals of parameters $x$ and $y$ on the decision-making target $z$. These are $[181, 221]$ and $[190, 210]$, respectively. Therefore, the sensitivity factors $\theta^x$ and $\theta^y$ are 2/3 and 1/3, respectively; the sensitivity of parameter $x$ is greater than that of parameter $y$ for the same decision-making target. If $\delta_x = 0.1$ and $\delta_y = 0.3$, the sensitivity factors $\theta^x$ and $\theta^y$ are 4/9 and 5/9, respectively. Here, the sensitivity degree of parameter $x$ to the decision-making target $z$ is less than that of parameter $y$. If $\delta_x = 0.01$, the same conclusion, $\delta_x = 0.1$, can be made. It can be stated that different coefficients of variation can result in different sensitivity factors of the model parameters to the decision-making target. When the same coefficients of variation are used, the same sensitivity factors are desired.

Thus, to obtain the sensitivity factor matrix $\theta^y$, the parameter intervals must be determined using the same coefficient of variation $\delta$ and parameter medium-value $X^c$.

4 Iterative optimization of the inverse analysis model

For complex nonlinear problems, when the forward convergence condition is not satisfied, the interval reversible inverse analysis model does not work well.

Considering the uncertainty, the unknown parameter vector is denoted as $\alpha^1$. For practical engineering problems, a loose parameter vector $\alpha^i$ can be set by engineering experience, testing information, exploration data, and so on. This may ensure that the solution of the inverse problem is in accordance with actual physical concepts and geological data. If the measured displacement is taken to be $U^i$, the following inverse analysis model can be proposed for the interval parameter vector $\alpha^i$ by the optimization method:

\[
\begin{align*}
\text{for} & \quad \min / \max \alpha_i \ (i = 1, 2, \cdots, m) \\
\text{s.t.} & \quad K(\alpha, U) U = R(\alpha) \\
& \quad \alpha \in \alpha^i \\
& \quad U_{\text{cal}} \in U^i
\end{align*}
\]

where $U_{\text{cal}}$ is the calculated displacement by $K(\alpha, U) U = R(\alpha)$. Therefore, we have $\alpha^i = [\min(\alpha_1), \max(\alpha_1), \min(\alpha_2), \max(\alpha_2), \cdots, \min(\alpha_m), \max(\alpha_m)]^T$ (16)

Equation (15) is sensitive to the initial values of the parameters. For comparison, different initial values are used [6]. Thus, an iterative optimization of the inverse analysis model is proposed as follows:

(1) Step 1. Select one initial parameter vector $\alpha_0$ ($\alpha_0 \in \alpha^i$). Here, $U_{\text{cal}} \in U^i$ and $U_{\text{cal}}$ are computed by $K(\alpha_0, U) U = R(\alpha_0)$.

(2) Step 2. Compute $\alpha_i^0$ by the following optimization method:
\[
\begin{align*}
\text{for} & \quad \min / \max \alpha_{i0} \quad (i = 1, 2, \ldots, m) \\
\text{s.t.} & \quad K(\mathbf{\alpha}, U)U = R(\mathbf{\alpha}) \quad (17) \\
& \quad \alpha_{i0} \in \alpha_{i0}^1 \\
& \quad \alpha_{i0} = \alpha_{k0} \quad (k = 1, 2, \ldots, m; \; k \neq i) \\
& \quad U_{cd} \in U_{t1}^1
\end{align*}
\]

Therefore, we have
\[
\mathbf{\alpha}_0^1 = \left[\begin{array}{c}
\text{[min}(\alpha_{i0}), \text{max}(\alpha_{i0})], \text{[min}(\alpha_{i0}), \text{max}(\alpha_{i0})], \cdots, \\
\text{[min}(\alpha_{i0}), \text{max}(\alpha_{i0})]\end{array}\right]^T
\]

(3) Step 3. Assume the parameter vector of step \(t\) is \(\mathbf{\alpha}_t^1\), the parameter vector of step \(t+1\) can be gained by

\[
\begin{align*}
\text{for} & \quad \min / \max \alpha_{i(t+1)} \quad (i = 1, 2, \ldots, m) \\
\text{s.t.} & \quad K(\mathbf{\alpha}_{t+1}, U)U = R(\mathbf{\alpha}_{t+1}) \\
& \quad \alpha_{i(t+1)} \in \alpha_{i(t+1)}^1 \\
& \quad \alpha_{k(t+1)} \in \alpha_{i(t+1)}^1 \quad (k = 1, 2, \ldots, m; \; k \neq i) \\
& \quad U_{cd} \in U_{t1}^1
\end{align*}
\]

Therefore, we have
\[
\mathbf{\alpha}_{t+1}^1 = \left[\begin{array}{c}
\text{[min}(\alpha_{i(t+1)}), \text{max}(\alpha_{i(t+1)})], \\
\text{[min}(\alpha_{i(t+1)}), \text{max}(\alpha_{i(t+1)})], \cdots, \\
\text{[min}(\alpha_{i(t+1)}), \text{max}(\alpha_{i(t+1)})]\end{array}\right]^T
\]

(4) Step 4. After obtaining the final parameter vectors by an iterative optimization process, the convergence condition of the iterative optimization can be defined by the following vector norm:
\[
\left\| \mathbf{\alpha}_{t+1}^1 - \mathbf{\alpha}_t^1 \right\| < \varepsilon \quad (21)
\]

where \(\|\cdot\|\) is the interval vector norm, and \(\varepsilon\) is a small parameter. If the norm is a Euclidean norm, then
\[
\left\| \mathbf{\alpha}_{t+1}^1 - \mathbf{\alpha}_t^1 \right\| = \sqrt{\left\| \mathbf{\alpha}_{t+1}^1 - \mathbf{\alpha}_t^1 \right\|^2 + \left\| \mathbf{\alpha}_{t+1}^1 - \mathbf{\alpha}_t^1 \right\|^2} \quad (22)
\]

During the process of convergence, the final intervals of unknown parameters gradually approach the exact results. So the process is actually a naturally converging one. Genetic algorithms have powerful capabilities in global searches and limited capabilities in local searches; however, simulated annealing algorithms have powerful local search capabilities and limited capabilities in global searches. Therefore, for rapid and correct convergence, genetic and simulated annealing algorithms are used together for the optimization.

5 Numerical example

To demonstrate the applications of the proposed method, and to estimate the effect of measured precisions on analysis results, a tunnel excavation example is considered.

The simulation involves a tunnel with a horseshoe cross-section that is 7.8 m wide and 9.0 m high. The intervals of unknown parameters (cohesion force \(c\), friction angle \(\phi\), Young’s modulus \(E\) and Poisson’s ratio \(\nu\)) are analyzed using the iterative optimization of an inverse analysis model. The computation zone is divided into a mesh consisting of 488 plane strain elements and 485 nodes (Fig.1); and the boundaries have normal constrains. A Drucker-Prager constitutive model is employed to simulate rock behavior.

![Fig.1 The finite element meshes.](image)

According to the geological exploration data, field and laboratory tests, and expert opinion, the loose intervals of unknown parameters were adopted as follows: \(0.2 \; \text{MPa} \leq c \leq 0.7 \; \text{MPa}, \; 27^\circ \leq \phi \leq 39^\circ, \; 1.3 \; \text{GPa} \leq E \leq 6.0 \; \text{GPa}, \; 0.30 \leq \nu \leq 0.35\).

The transducers and measuring lines around the tunnel are shown in Fig.2. The central value vector of the parameters \(\{c, \phi, E, \nu\} = \{0.45 \; \text{MPa}, \; 33^\circ, \; 3.65 \; \text{GPa}, \; 0.325\}\). The displacements of the measuring lines are shown in Table 1.

The coefficient of variation is taken as 0.1. The boundary influence value intervals and the decision-
Fig. 2 Transducers and measuring lines around the tunnel.

Table 1 The displacements of various measuring lines.

<table>
<thead>
<tr>
<th>Measuring line</th>
<th>Displacement (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>2.306</td>
</tr>
<tr>
<td>CD</td>
<td>3.512</td>
</tr>
<tr>
<td>AD</td>
<td>5.699</td>
</tr>
<tr>
<td>CE</td>
<td>3.672</td>
</tr>
</tbody>
</table>

Table 2 The boundary influence value intervals \([y_i^*, y_j^*]\). mm

<table>
<thead>
<tr>
<th>(D_{AB})</th>
<th>(D_{CD})</th>
<th>(D_{AD})</th>
<th>(D_{CE})</th>
</tr>
</thead>
<tbody>
<tr>
<td>[2.248, 2.385]</td>
<td>[3.434, 3.616]</td>
<td>[5.651, 5.766]</td>
<td>[3.448, 3.961]</td>
</tr>
<tr>
<td>[2.218, 2.423]</td>
<td>[3.398, 3.662]</td>
<td>[5.653, 5.707]</td>
<td>[3.350, 4.093]</td>
</tr>
<tr>
<td>[2.097, 2.563]</td>
<td>[3.193, 3.903]</td>
<td>[5.181, 6.332]</td>
<td>[3.338, 4.080]</td>
</tr>
<tr>
<td>[2.185, 2.433]</td>
<td>[3.398, 3.636]</td>
<td>[5.566, 5.857]</td>
<td>[3.241, 4.093]</td>
</tr>
</tbody>
</table>

Note that the column vectors represent the influence factors of \(\varphi\), \(c\), \(E\) and \(\nu\) on the displacements of measuring lines \(AB\), \(CD\), \(AD\) and \(CE\), respectively. On the contrary, the row vectors show the influence of the measuring lines \(AB\), \(CD\), \(AD\) and \(CE\) on the parameters \(\varphi\), \(c\), \(E\) and \(\nu\), respectively. The parameters \(E\) and \(c\) are obviously more sensitive to the displacements along the measuring lines than the parameters \(\nu\) and \(\varphi\).

For different man-made errors and implementation precisions, the measurement precisions are assumed to be 0.05 and 0.10 mm, respectively. The parameters \(c\), \(\varphi\), \(E\) and \(\nu\), or \(c\) and \(E\) are, respectively, assumed to be unknown. The upper and lower bounds of the unknown parameters are shown in Table 4.

From identification methods 1 and 3 or 2 and 4, the ranges of the parameters calculated by the proposed iteratively optimized inverse analysis method are clearly wider than those calculated by the directly optimized inverse analysis method. Therefore, the proposed method can result in more accurate results that agree well with the measured data. From identification methods 1–4, we can see sharper intervals of unknown parameters that are precisely measured. With the same measurements, we can obtain sharper interval ranges for the few unknown parameters. Due to the lower sensitivity, the parameter intervals obtained for \(\varphi\) and \(\nu\) may reach the given loose intervals.

Table 3 The decision-making target intervals \([y_i^*, y_j^*]\). mm

<table>
<thead>
<tr>
<th>(D_{AB})</th>
<th>(D_{CD})</th>
<th>(D_{AD})</th>
<th>(D_{CE})</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1.864, 2.944]</td>
<td>[2.922, 4.338]</td>
<td>[5.000, 6.691]</td>
<td>[2.494, 5.408]</td>
</tr>
</tbody>
</table>

Table 4 The identification results for unknown parameters.

<table>
<thead>
<tr>
<th>Identification method No.</th>
<th>Measurement precision (mm)</th>
<th>(c) (MPa)</th>
<th>(\varphi) (°)</th>
<th>(E) (GPa)</th>
<th>(\nu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.05</td>
<td>0.416</td>
<td>0.475</td>
<td>29.569</td>
<td>36.107</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>0.383</td>
<td>0.524</td>
<td>27.000</td>
<td>39.000</td>
</tr>
<tr>
<td>2</td>
<td>0.05</td>
<td>0.442</td>
<td>0.461</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>0.433</td>
<td>0.471</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>3</td>
<td>0.05</td>
<td>0.436</td>
<td>0.468</td>
<td>30.893</td>
<td>35.018</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>0.423</td>
<td>0.489</td>
<td>28.932</td>
<td>36.875</td>
</tr>
<tr>
<td>4</td>
<td>0.05</td>
<td>0.440</td>
<td>0.460</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>0.432</td>
<td>0.472</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

Note: For identification methods 1 and 2, the iteratively optimized inverse analysis method is used; for identification methods 3 and 4, the directly optimized inverse analysis method is used. For identification methods 1 and 3, the unknown parameters are \(c\), \(\varphi\), \(E\) and \(\nu\); for identification methods 2 and 4, the unknown parameters are \(c\) and \(E\).
6 Conclusions

Uncertainty in engineering can be described by interval mathematics. In this paper, a new sensitivity analysis method for engineering parameters is proposed in the form of an interval analysis method, thereby extending the application domain of the interval analysis method. The purpose of sensitivity analysis in engineering applications is primarily directed towards the optimization of structural design and controlling testing or construction quality.

The intervals of unknown parameters can be obtained using an interval inverse analysis method based on the measured data. For complex nonlinear problems, an interval iteratively optimized inverse analysis model is proposed. All of the parameters’ intervals that agree well with the measured data can be determined by the method using given loosely-constrained intervals. Here, point data corresponding to parameter intervals determined by the inverse analysis method are slightly wider in range than the measured data. Unlike a traditional inverse analysis model, an iteratively optimized inverse analysis model can result in a unique solution, thereby solving one of the outstanding problems in current inverse problems.

References


